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Scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences (14 May 1997)

A scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences was held on 14 May 1997 at the P L Kapitza Institute for Physical Problems, RAS. The following reports were presented at the session:

(1) **Mineev V P, Vavilov M G** (Landau Institute of Theoretical Physics, RAS, Chernogolovka, Moscow Region) "De Haas – van Alphen effect in superconductors";

(2) **Volkov V A, Takhtamirov É E** (Institute of Radio-engineering and Electronics, RAS, Moscow) "Dynamics of an electron with space-dependent mass and the effective-mass method for semiconductor heterostructures";

(3) **Sukhorukov A P** (M V Lomonosov Moscow State University, Moscow) "New avenue of investigation in the physics of solitons: parametrically-coupled solitons in a quadratically-nonlinear medium";

(4) **Bogatov A P** (P N Lebedev Physics Institute, RAS, Moscow) "Optics of semiconductor lasers";

(5) **Korovin S D** (Institute of High-Power Electronics, Tomsk) "Generation of high-power microwave radiation on the base of high-current nanosecond electron beams";

(6) **Ardelyan N V, Bisnovatyĭ-Kogan G S, Moiseenko S G** (M V Lomonosov Moscow State University, Moscow; Institute of Space Research, Moscow) "Explosion mechanisms of supernovae: the magnetorotational model";

(7) **Slysh V I** (Astrocosmic Centre of the P N Lebedev Physics Institute, RAS, Moscow) "Stars, planets, and cosmic masers".

Summaries of four (1, 2, 6, 7) of the reports are given below.

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De Haas – van Alphen effect in superconductors

V P Mineev, M G Vavilov

The quantum oscillations of magnetization or, alternatively, the de Haas – van Alphen (dHvA) effect, is a well-studied phenomenon in the physics of normal metals. According to the universally accepted Lifshitz – Kosevich theory [1], each extreme section of the Fermi surface contributes to the oscillating part of the magnetization to yield

$$M_{\rm osc} \sim \sqrt{H}\,\frac{2\pi^2 T/\omega_{\rm c}}{\sinh(2\pi^2 T/\omega_{\rm c})}\exp\left(-\frac{\pi}{\omega_{\rm c}\tau}\right)\sin\left(\frac{2\pi F}{H}+\Phi\right). \quad (1)$$

Here $\omega_{\rm c} = eH/m^*c$ is the cyclotron frequency, $F = cS/2\pi e$, S is the area of the extreme section of the Fermi surface, and τ is the electron-vacancy scattering time. The Planck constant $\hbar$ is presumed hereafter to be equal to 1. The quantity $1/2\pi\tau$ is commonly referred as the Dingle temperature. Both the temperature and impurity-dependent factors in formula (1) fall rapidly with decreasing magnetic field, so that in normal metals dHvA oscillations are observable in sufficiently high fields. In the case of superconductors, the magnetic fields in which the dHvA effect is accessible to observation usually far exceed the critical field of the superconducting-normal phase transition. Therefore, the marked oscillations of magnetization would be expected to appear only in the region of very low temperatures

$$T < \frac{eH_{\rm c2}}{2\pi^2 m^* c} \sim \frac{T_{\rm c}^2}{\mu}\,. \quad (2)$$

Here $T_{\rm c}$ is the transition temperature in a zero magnetic field, and μ is the Fermi energy. On the other hand, because of electron-vacancy scattering [2], dHvA oscillations come into prominence only in sufficiently pure metals, i.e. when the condition $\omega_{\rm c}\tau \gg 1$ or, equivalently, $l_{\rm imp} \gg R_{\rm c}$, is fulfilled. Here $l_{\rm imp} = v_{\rm f}\tau$ is the mean free path, and $R_{\rm c} = k_{\rm f}\lambda^2$ is the cyclotron radius with $k_{\rm f}$ being the Fermi wave vector and $\lambda = \sqrt{c/eH}$ the magnetic length. In magnetic fields of the order of $H_{\rm c2}$, the quantity λ coincides with the coherence length $\xi(T)$. Therefore, the requirements on the sample purity which would be sufficient to observe dHvA oscillations in fields of the order of $H_{\rm c2}$, viz.

$$l_{\rm imp} \gg k_{\rm f}\xi^2\,, \quad (3)$$

is much more stringent than the conventional condition on semiconductor purity $l_{\rm imp} \gg \xi_0$.

Thus, observations of the dHvA effect in the regions of fields and temperatures typical of type II superconductors are possible only in the case of ultra-pure superconductors with a large magnitude of the upper critical field, which are rather rare. Among these are compounds with the A-15 structure (V_3Si, Nb_3Sn) [3, 4], boron carbides ($YNiB_2C$) [5] as well as some organic and layered superconductors (see the reviews [6, 7]). For example, in V_3Si [3] where $H_{\rm c2} = 18.5$ T, $T_{\rm c} = 17$ K, $\xi_0 = 6.3$ nm and $l_{\rm imp} > R_{\rm c} = 130$ nm, dHvA oscillations in fields of the order of $H_{\rm c2}$ are accessible to observation at temperatures of the order of 1 K.

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The dHvA effect in these substances remains when going to the mixed state ($H < H_{c2}$). In this state, the oscillation frequency does not change, whereas the amplitude falls with decreasing field more rapidly than in the normal state.

The suppression of the magnetization-oscillation amplitude in type II superconductors was calculated in the theoretical works [8] and [9]. It was shown that quasi-particle scattering by the nonuniform spatial distribution of the order parameter $\Delta(\mathbf{R})$ in the mixed state results in an additional broadening of the Landau levels:

$$\frac{1}{\tau_s} \sim \sqrt{\mu\omega_c}\,\frac{H_{c2} - H}{H_{c2}} . \tag{4}$$

As a result, the amplitude of the dHvA effect attains, apart from the Dingle factor, another temperature-independent multiplier $\exp(-\pi/\omega_c\tau_s)$ and decreases rather rapidly away from the phase transition line H_{c2}.

The derivation of expression (4) is inadequate from the theoretical standpoint. The matter is that the electron spectrum and the level broadening were obtained by Maki [8] through formally replacing the quasi-classical spectrum found by Brandt et al. [10] with the corresponding quantum expression. The quasi-classical description in terms of the continuous variables $\xi = k^2/2m - \mu$ and polar angle θ is quite adequate when the distance between the Landau levels is small in comparison with temperature T or level width $\Gamma = 1/2\tau$. When studying the dHvA effect, we are dealing with the opposite situation $\omega_c > 2\pi^2 T$ and $\omega_c > \pi\Gamma$, so that the quasi-classical approach is inapplicable for calculation of the spectrum.

Nevertheless, the quantum approach developed by Stephen [9] has strengthened the results by Maki [8]. However, when calculating the quasi-particle-energy eigenvalue in the limit of low temperatures $T < \omega_c$ [9], the summation over the principle quantum number was replaced with the integration that is admissible only in the case when the width of the levels exceeds the distance between them. That was the reason why the results by Stephen [9] and Maki [8] turned out the same.

Some other descriptions of the dHvA effect in superconductors were also proposed [11–13]. Different approaches were put forward, but in one way or another the BCS-type spectrum was used, namely

$$E = \sqrt{E_n^2(k_z) + \Delta^2} , \tag{5}$$

where

$$E_n(k_z) = \omega_c\left(n + \frac{1}{2}\right) + \frac{k_z^2}{2m} - \mu . \tag{6}$$

Stephen [9] showed that spectrum (5) is realized only in sufficiently weak fields $\sqrt{\mu\omega_c} \ll T$, therefore, in view of Eqn (2), magnetization oscillations in this region are not observable.

Formally, spectrum (5) is also derived in the ultra-quantum limit $\omega_c \sim \mu$ [14]. However, it is known that in the ultra-quantum limit the mean-field approximation in the theory of superconductivity is inapplicable (see Ref. [15]) and, thus, the mathematical model used in Ref. [14] does not provide an adequate description of superconductivity in strong magnetic fields.

We developed a self-consistent quantum theory of the dHvA effect in the mixed state [16]. It was shown that at a finite concentration of impurities, despite the requirement of high purity $\omega_c > \pi\Gamma$ that is necessary to observe the dHvA effect, in the mixed state near the upper critical field H_{c2} a region of gapless superconductivity exists, in which the density of states on the Fermi surface remains finite, i.e.

$$N(E = 0) = N_0\left(1 - \frac{2\sqrt{\pi^3 n_f}}{L\ln n_f}\,\frac{H_{c2} - H}{H_{c2}}\right) . \tag{7}$$

Here N_0 is the density of states in the normal metal, $n_f = \mu/\omega_c$, and L is a numerical constant, $L \approx 2$.

According to Eqn (7), in the vicinity of the metal–superconductor phase transition there is a region of gapless superconductivity in the field range

$$\frac{H_{c2} - H}{H_{c2}} < \frac{\ln n_f}{\pi^{3/2}\sqrt{n_f}} . \tag{8}$$

For the most superconductors the number n_f is rather large. Because of this, the gapless superconductivity is realized in a very narrow range of magnetic fields. Nevertheless, n_f does not exceed 50 for superconductors in which dHvA oscillations were observed [3, 4]. In accordance with Eqn (8), the density of states at the Fermi level remains finite at $H_{c2} - H \approx 0.1 H_{c2}$.

The oscillating part of the density of states at the Fermi level and hence the oscillating part of the magnetization M_s^{osc} is also suppressed in the mixed state relative to its value M_n^{osc} in the normal state:

$$\frac{M_s^{osc}}{M_n^{osc}} = 1 - \frac{2\sqrt{\pi n_f}}{L\ln n_f}\,\frac{H_{c2} - H}{H_{c2}} . \tag{9}$$

Expressions (7) and (9) were obtained in the linear approximation with respect to the square of the order parameter $\Delta^2 \sim (H_{c2} - H)/H_{c2}$ with the proviso that $T < \Gamma \ll \omega_c$. Going beyond the scope of the linear approximation presents serious mathematical problems resulting from the nondiagonality of the energy-eigenvalue matrix, which necessarily occurs because of the irregular spatial distribution of the order parameter.

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