

Influence of interaction on weak localization.

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We discuss the influence of the electromagnetic environment and the electron–electron interaction on the weak localization correction to the conductivity of a disordered metal. The theory of this phenomenon for sufficiently high temperature, where the quantum nature of the interaction of electrons with the electromagnetic field can be disregarded, has been understood for some time. We consider the first order quantum correction to this semiclassical description and work out the temperature range in which this correction is small. No external low frequency cut–off is needed in our calculation. We conclude that in the whole region of temperature where the weak localization correction is much smaller than the Drude conductivity the classical treatment of the interaction is valid.

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I. INTRODUCTION

Experimental and theoretical studies of weak localization have given considerable insight into the physics of small disordered conductors. A review of this research can be found in [1]. Since interference between time-reversed electron trajectories [1,2] is the root cause of weak localization, its strength depends on phase coherence between such paths. Dephasing can be due to extrinsic causes, such as applied magnetic fields, or to intrinsic mechanisms that remove phase information, for example scattering against localized spins, or the electron-phonon and electron-electron interactions, the last being the more important at low temperatures.

In this paper we reconsider the effect of electron-electron interactions on weak localization. Our aim is to understand quantum corrections to a theory [3] in which the influence of these interactions is modeled as a fluctuating classical electromagnetic field. The subject is topical because of experimental work [4] suggesting that the dephasing rate saturates to a finite value as the temperature approaches zero, whereas ref. [3] predicts the rate to vanish in this limit. Now, one certainly expects quantum corrections to the last mentioned theory. The relevant question, which we address here, is whether the corrections are important in the region of weak localization. Our conclusion, in brief, is that they are not.

It is worth comparing our approach to quantum effects with a recent preprint [5] in which very complete calculations of the quantum mechanical correction to the Drude conductivity of a weakly disordered metal are carried out to second order in the screened electron-electron interaction. In the absence of dephasing, the weak localization correction for narrow wires contains an infrared divergence. This divergence cannot be cured in finite order perturbation theory. For this reason, a magnetic field is posited in ref. [5], providing a low frequency cut-off. The cut-off dependence of the correction to weak localization is found to be exactly as expected from [3]. Although we find this argument completely convincing, it is nonetheless true that the results of ref. [5] are, strictly speaking, only valid when the electron-electron interaction produces a small correction to the effect due to some other mechanism. [See, however, Section VI, below.] Now, the calculation of [3] has the considerable merit of treating classical fluctuations exactly, thereby being free of low frequency divergences. By building on this calculation, we are able to test its validity in the absence of extrinsic influences.

According to paper [5], the first order diagrams in the interaction can be divided into two parts: $\Delta\sigma_{wl}^{(1)} = \Delta\sigma_{deph} + \Delta\sigma_{cwl}$, $\Delta\sigma_{deph}$ is called the dephasing term and the other $\Delta\sigma_{cwl}$ is the interaction correction to weak localization. Although they both originate from the same type of diagrams they have different dependences on the parameters of the system. For example, it was shown in [5], that for one dimensional wires

$$\begin{aligned}\Delta\sigma_{deph} &= \frac{e^2\sqrt{D\tau_H}}{\pi\hbar} D^{1/2}\tau_H^{3/2} \frac{e^2T}{4\hbar^2\sigma_1} \\ \Delta\sigma_{cwl} &= -3\zeta(3/2) \frac{e^2\sqrt{D\tau_H}}{2\pi\hbar} \frac{e^2}{2\pi\hbar\sigma_1} \sqrt{\frac{\hbar D}{2\pi T}}.\end{aligned}\tag{1}$$

Here τ_H is the dephasing time due to an external magnetic field, which appears in the calculations of [5] as a low frequency cutoff. We see that the most singular term is the dephasing correction which is proportional to τ_H^2 , while $\Delta\sigma_{cwl} \sim \sqrt{\tau_H}$. We consider here only the dephasing term of the weak localization correction to conductivity.

The physical meaning of both of these terms was explained in [5]. We would like to note only that the dephasing term is the result of the interference of two time reversed paths (see below), while the interaction correction term is the result of interference for more complicated trajectories.

It is appropriate here to mention some other recent work. In e-print [6] by us we suggested that the saturation in dephasing rate [4] can be explained in the framework of the Caldeira–Leggett model [7]. This paper is simply wrong, because it treats the phase as a single particle and loses the physics associated with the exclusion principle. The present work is the promised revision of ref. [6]. We also remark that Golubev and Zaikin [8] have a calculation which claims to treat interaction effects to all orders in perturbation theory and obtains a finite dephasing rate from the electron–electron interaction at zero temperature. We believe that this surprising result is due to uncontrolled approximations, and comment on it is in Appendix C of this paper.

In outline, the plan of this paper is as follows. In the next section we introduce a model in which the dephasing environment is modeled by a set of harmonic oscillators, in the manner of Feynman and Vernon [9] and Leggett and Caldeira [7], which are coupled to the charge density of an electron gas in a random potential. By a suitable choice of the spectrum of oscillators, we obtain after integrating out the oscillators the influence functional corresponding to the diffusively screened electron interaction. Up to this point our calculation is identical to that in refs. [5] and [8]. We deviate from previous work by now separating the influence functional into a classical part and a remainder. In section 3 we show that the effect of the classical fluctuations on the back scattering (or weak localization) correction to the Drude conductivity can be written as a path integral, thereby exactly reproducing the result obtained in ref. [3]. The improvement over that work is that we have an explicit expression for the remainder. At this point it is convenient to specialize the model to the case of Nyquist noise which yields a simpler path integral, and allows further calculations to be done analytically. This model is used in Section 4 to explicitly calculate the quantum corrections to second order. In Section 5 we give semiquantitative arguments for how these calculations are modified when the more realistic spectrum corresponding to the screened electron–electron interaction is used, and infer the structure of the corresponding quantum corrections up to numerical coefficients. The final section contains a discussion of the results and conclusions.

II. INFLUENCE FUNCTIONAL METHOD FOR A DISORDERED INTERACTING ELECTRON GAS

As the starting point for our calculations we use a variant of the Feynman-Vernon [9], Leggett-Caldeira [7] method in which a dissipative environment is described by a set of harmonic degrees of freedom. In this way we are able to construct a formalism general enough to accommodate different models for the electron–electron interaction.

Consider a closed system described by a Hamiltonian consisting of three parts

$$\mathcal{H}(t) = \mathcal{H}_0(t) + \mathcal{H}_{env}(t) + \mathcal{H}_{int}(t). \quad (2)$$

Here the first term describes a disordered free electron subsystem,

$$\mathcal{H}_0(t) = \int d\mathbf{r} \psi^\dagger(t, \mathbf{r}) \left(-i \frac{\partial}{\partial t} - \frac{\nabla^2}{2m} - \mu + U(r) \right) \psi(t, \mathbf{r}), \quad (3)$$

where μ is chemical potential and $U(\mathbf{r})$ is a random impurity potential. The next term is the Hamiltonian for harmonic electromagnetic modes,

$$\mathcal{H}_{env}(t) = \sum_{\nu} \omega_{\nu} \left(a_{\nu}^{\dagger} a_{\nu} + \frac{1}{2} \right), \quad (4)$$

where ν labels different modes of the electric field, which correspond to spatial wave functions $\phi_{\nu}(\mathbf{r})$ which can be chosen to be real. The interaction between the field and the electron system is

$$\mathcal{H}_{int}(t) = \int d\mathbf{r} \psi^\dagger(t, \mathbf{r}) v(t, \mathbf{r}) \psi(t, \mathbf{r}) = \int d\mathbf{r} v(t, \mathbf{r}) q(t, \mathbf{r}), \quad (5)$$

where $q(t, \mathbf{r}) = \psi^\dagger(t, \mathbf{r}) \psi(t, \mathbf{r})$ and

$$v(t, \mathbf{r}) = \sum_{\nu} \frac{e}{\sqrt{2M_{\nu}\omega_{\nu}}} (\phi_{\nu}(\mathbf{r}) a_{\nu}^{\dagger}(t) + \phi_{\nu}^*(\mathbf{r}) a_{\nu}(t)) \quad (6)$$

is an operator in the space of the electric field quantum states. Above, the Fermion and Boson field operators, $\psi(t, \mathbf{r})$ and $a_\nu(t)$, are in the Heisenberg representation.

The electron Green's function is defined according to

$$G(t, \mathbf{r}, t', \mathbf{r}') = \begin{cases} -i\langle \psi(t, \mathbf{r})\psi^+(t', \mathbf{r}') \rangle, & \text{if } t >_K t', \\ i\langle \psi^+(t', \mathbf{r}')\psi(t, \mathbf{r}) \rangle, & \text{if } t <_K t'. \end{cases} \quad (7)$$

Here the average $\langle \dots \rangle$ is understood as a trace over the quantum state of the whole system, taken with the density matrix ρ of the system:

$$\langle \dots \rangle = \frac{\text{Tr}(\rho T_K(\dots \exp(iS[\psi^+, \psi, V])))}{\text{Tr}(\rho T_K(\exp(iS[\psi^+, \psi, V])))}, \quad (8)$$

The action introduced in Eq.(8) has the form:

$$S[\psi^+, \psi, V] = - \int_K dt (\mathcal{H}_0 + \mathcal{H}_{env} + \mathcal{H}_{int}). \quad (9)$$

In the above formulas K refers to the Keldysh contour, which runs from $-\infty$ to $+\infty$ and then backward to $-\infty$, (see, e.g. [10]), and the subscript K refers to ordering along this contour. With no loss of generality we may take the $t = -\infty$ initial thermal state of the system to be a product of the noninteracting electron density matrix ρ_{el} and the environment density matrix ρ_{env} , where ρ_{el} obeys Fermi-Dirac statistics and ρ_{env} obeys Bose-Einstein statistics

Since the electric field is described by a set of Harmonic oscillators, the trace over its quantum states can be worked out exactly. This yields an influence functional [cf. ref [9]] for the electrons:

$$F[q, q'] = \exp(-\Phi[q, q']), \quad (10)$$

with

$$\begin{aligned} \Phi[q, q'] = & \frac{1}{2} \int_{-\infty}^{+\infty} ds \int_{-\infty}^{+\infty} du \int d\mathbf{r}_1 \int d\mathbf{r}_2 \\ & ((q(s, \mathbf{r}_1) - q'(s, \mathbf{r}_1))\mathcal{K}_1(s - u, \mathbf{r}_1, \mathbf{r}_2)(q(u, \mathbf{r}_2) - q'(u, \mathbf{r}_2)) + \\ & (q(s, \mathbf{r}_1) - q'(s, \mathbf{r}_1))\mathcal{K}_2(s - u, \mathbf{r}_1, \mathbf{r}_2)(q(u, \mathbf{r}_2) + q'(u, \mathbf{r}_2))) \end{aligned} \quad (11)$$

where we have introduced the notations

$$\mathcal{K}_1(s - u, \mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} \sum_{\nu} \frac{e^2}{2M_{\nu}\omega_{\nu}} \coth \frac{\omega_{\nu}}{2T} \cos(\omega_{\nu}(s - u))\phi_{\nu}(\mathbf{r}_1)\phi_{\nu}(\mathbf{r}_2) \quad (12)$$

and

$$\mathcal{K}_2(s - u, \mathbf{r}_1, \mathbf{r}_2) = -i\Theta(s - u) \sum_{\nu} \frac{e^2}{2M_{\nu}\omega_{\nu}} \sin(\omega_{\nu}(s - u))\phi_{\nu}(\mathbf{r}_1)\phi_{\nu}(\mathbf{r}_2). \quad (13)$$

Above $q(s, \mathbf{r}) = \psi^+(s, \mathbf{r})\psi(s, \mathbf{r})$ is the electron density taken at the forward part of the Keldysh contour, and $q'(s, \mathbf{r}) = \psi^+(s, \mathbf{r})\psi(s, \mathbf{r})$ is taken at the backward part of the contour.

Note that a result of the very same form can be obtained for the Coulomb interaction

$$\mathcal{H}_{int}(t) = \int \int d\mathbf{r}d\mathbf{r}' q(t, \mathbf{r})V(\mathbf{r} - \mathbf{r}')q(t, \mathbf{r}'), \quad (14)$$

where $V(\mathbf{r}) = e^2/|\mathbf{r}|$. The usual way to achieve this can be found in, e.g., [11]. One performs the Hubbard–Stratanovich transformation to decouple the density operators $q(t, \mathbf{r}')$ in Eq.(14), introducing a fluctuating electric field. Then the effective action is expanded to second order in the fluctuating fields, and the screened random phase approximation is used for the electronic polarization. Finally one integrates over the Hubbard–Stratanovich variables to obtain an effective action for electrons in the form of Eq.(10), with the specific forms of \mathcal{K}_1 and \mathcal{K}_2 given by the choice of mode density in Eq.(26) below.

The influence functional Eq.(11) is very similar to that obtained for a quantum particle coupled to the environment of harmonic oscillators. Here, however, $q(s, \mathbf{r})$ is not a particle coordinate but fermion density operator. Expanding the exponent of the influence functional $F[q, q']$ we can reproduce the Keldysh diagram technique for electrons coupled to

a Bose field with a propagator, which can be expressed in terms of \mathcal{K}_1 and \mathcal{K}_2 . One finds that the Keldysh component of the Bose field is \mathcal{K}_1 , and the retarded component is \mathcal{K}_2 . It is convenient to use the electronic Keldysh Green's function in the 'rotated' form [10]

$$\hat{G}(t, \mathbf{r}, t', \mathbf{r}') = \begin{pmatrix} G^{(R)}(t, \mathbf{r}, t', \mathbf{r}') & G^{(K)}(t, \mathbf{r}, t', \mathbf{r}') \\ 0 & G^{(A)}(t, \mathbf{r}, t', \mathbf{r}') \end{pmatrix}. \quad (15)$$

One can check that in this representation the vertices corresponding to the the coupling of electrons via \mathcal{K}_1 and the vertex at time s of the electron coupling via \mathcal{K}_2 in Eq.(11) are proportional to the unit matrix in the Keldysh space, whereas the vertex corresponding to the electron scattering by the field $\mathcal{K}_2(s - u, \mathbf{r}_1, \mathbf{r}_2)$ at time u is proportional to the first Pauli matrix τ_x , because of the plus sign between q and q' at this vertex in Eq. (11).

The structure achieved thus far is formally identical to the starting points of refs. [5] and [8]. At this stage, we make a new departure by explicitly distinguishing between the classical and quantum effects of the 'environment. Note that at high temperature \mathcal{K}_1 contains the huge factor $\coth \omega/2T \approx 2T/\omega$, so that the second term \mathcal{K}_2 is small by comparison. This fact allows us to represent the effective action $F[q, q']$ as a product of classical $F_c[q, q']$ and quantum $F_q[q, q']$ parts.

$$F[q, q'] = F_c[q, q']F_q[q, q']. \quad (16)$$

In F_q we include all of K_2 from Eq.(10) and the part of K_1 chosen as

$$\mathcal{K}_{1q}(s - u, \mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} \sum_{\nu} \frac{e^2}{2M_{\nu}\omega_{\nu}} \left(\coth \frac{\omega_{\nu}}{2T} - \frac{2T}{\omega_{\nu}} \right) \cos(\omega_{\nu}(s - u)) \phi_{\nu}(\mathbf{r}_1) \phi_{\nu}(\mathbf{r}_2). \quad (17)$$

The remainder of K_1 , namely Eq.(12) with $\coth \omega/2T$ replaced by $2T/\omega$ is included in F_c . Our strategy will be to treat F_c exactly, and F_q as a perturbation.

We now introduce a new Hubbard–Stratanovich transformation to decrease a power of the fermion operators in the exponent of the classical part of the influence functional and obtain

$$F_c[q, q'] = \int \mathcal{D}\mathbf{w} e^{-\sum_{\nu} w_{\nu}^2 e^{iS_c[q, q', \mathbf{w}]}} \quad (18)$$

where

$$S_c[q, q', \mathbf{w}] = 2 \int_K dt \int d\mathbf{r} \sum_{\nu} \frac{e}{\sqrt{2M_{\nu}\omega_{\nu}}} w_{\nu} q(t, \mathbf{r}) \sqrt{\frac{T}{2\omega_{\nu}}} \phi_{\nu}(\mathbf{r}) \cos(\omega_{\nu}t + \varphi_{\nu}) = \int_K dt \int d\mathbf{r} V(t, \mathbf{r}) q(t, \mathbf{r}), \quad (19)$$

$$V(t, \mathbf{r}) = 2e \sum_{\nu} \frac{w_{\nu}}{\sqrt{2M_{\nu}\omega_{\nu}}} \sqrt{\frac{T}{2\omega_{\nu}}} \phi_{\nu}(\mathbf{r}) \cos(\omega_{\nu}t + \varphi_{\nu}), \quad (20)$$

$\mathbf{w}_{\mu} = (w_{\mu}, \phi_{\mu})$ is a two component Hubbard–Stratanovich variable for each electromagnetic field mode ν and

$$\int \mathcal{D}\mathbf{w} \dots = \prod_{\nu} \int \frac{dw_{\nu}}{\pi} \dots \quad (21)$$

After these operations, the average in Eq.(8) reduces to

$$\langle \dots \rangle = \frac{\text{Tr} (\rho T_K (\dots F_c[\psi^+, \psi] F_q[\psi^+, \psi] \exp(iS_0[\psi^+, \psi])))}{\text{Tr} (\rho T_K (F_c[\psi^+, \psi] F_q[\psi^+, \psi] \exp(iS_0[\psi^+, \psi])))} \quad (22)$$

and $S_0[\psi^+, \psi] = \int_K dt \mathcal{H}_0$.

As we have already mentioned, we consider finite order perturbation theory in $F_q[\psi^+, \psi]$ keeping all orders in $F_c[\psi^+, \psi]$. For this purpose it is convenient to introduce classical Green's function $\hat{G}_c(t, t', \mathbf{r}, \mathbf{r}')$. It is still defined by Eq.(7) where the average is taken in the sense of Eq.(22) with $F_q[\psi^+, \psi] = 1$. In the Keldysh representation

$$\hat{G}_c(t, \mathbf{r}, t', \mathbf{r}') = \begin{pmatrix} G_c^{(R)}(t, \mathbf{r}, t', \mathbf{r}') & G_c^{(K)}(t, \mathbf{r}, t', \mathbf{r}') \\ 0 & G_c^{(A)}(t, \mathbf{r}, t', \mathbf{r}') \end{pmatrix} \quad (23)$$

and

$$\begin{aligned}
G_c^{(R)} &= G_0^{(R)} + G_0^{(R)} V G_0^{(R)} + \dots \\
G_c^{(K)} &= G_0^{(K)} + G_0^{(R)} V G_0^{(K)} + G_0^{(K)} V G_0^{(A)} + G_0^{(R)} V \dots G_0^{(R)} V G_0^{(K)} V G_0^{(A)} \dots V G_0^{(A)} + \dots \\
G_c^{(A)} &= G_0^{(A)} + G_0^{(A)} V G_0^{(A)} + \dots,
\end{aligned} \tag{24}$$

where \hat{G}_0 is the Green's function of electron in metal without interaction.

To conclude this section we discuss two physically motivated choices for the density of environmental modes. Instead of the summation over mode index ν we integrate over ω and q , where ω is a frequency and q a wave vector, and write

$$\sum_{\nu} \frac{e^2}{2M_{\nu}\omega_{\nu}} \mathcal{F}(\omega_{\nu}, q_{\nu}) = \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} J(\omega, q) \mathcal{F}(\omega, q). \tag{25}$$

The choice

$$J(\omega, q) = (2\pi)^2 \sum_{\nu} \frac{e^2}{2M_{\nu}\omega_{\nu}} \delta(\omega - \omega_{\nu}) \delta(q - q_{\nu}) = \frac{e^2}{\sigma_1} \frac{\omega}{q^2}, \tag{26}$$

corresponds to the low frequency and small momentum spectral function of the screened Coulomb interaction in a disordered metal, with σ_1 the one dimensional conductivity of the wire.

Both for its own interest and because it gives an analytically simpler structure, we shall also consider the dissipative effect of Nyquist noise associated with an external resistor. We note that the spectral function of the electric field responsible for the Nyquist noise can be described by Eq.(26) with $q < q_m = 2\pi\kappa/L$ with σ_1 having the dimension of a 1D conductivity, $\kappa \ll 1$. This choice of κ allows us to take the limit of a uniform electric field inside the wire, corresponding to the Nyquist noise at the terminals of the wire produced by external part of the electric circuit. To see this, calculate the average value of the voltage $U = v(L) - v(0)$ at the ends of the wire, where $v(x)$ is defined by Eq.(6):

$$\begin{aligned}
\langle U_{\omega}^2 \rangle &= \langle (v(0) - v(L))_{\omega}^2 \rangle = \\
2e^2 \int_0^{q_m} (1 - \cos(qL)) &\frac{\omega}{\sigma_1} \frac{1}{q^2} \frac{dq}{2\pi} \coth \frac{\omega}{2T} \approx e^2 \omega R_{eff} \coth \frac{\omega}{2T}.
\end{aligned} \tag{27}$$

Here $R_{eff} = \kappa L / \sigma_1$ is now to be thought of as determined by resistances of the circuit. When R_{eff} is thus defined we see that the voltage fluctuations obey the fluctuation–dissipation theorem, see [12], appropriate to Nyquist noise.

For a wire with resistance R_w connected with resistor R_0 one has, see [3]

$$R_{eff} = \frac{R_w^2 R_0}{(R_0 + R_w)^2}. \tag{28}$$

III. CLASSICAL RESULT FOR THE WEAK LOCALIZATION CORRECTION TO THE CONDUCTIVITY

In this section we calculate the weak localization correction to the conductivity including the classical part of the influence functional, defined by Eqs. (2), exactly but neglecting the quantum part. We shall show that the environment now behaves like a classical electromagnetic field, so that the calculation—repeated here for completeness—is very similar to the one done in [3]. A small difference is that in our representation the fluctuating field is described by a scalar potential $V(t, \mathbf{r})$ instead of a vector potential.

The current operator is given by

$$\hat{\mathbf{j}}(\mathbf{r}) = \frac{ie}{2m} (\nabla_{\mathbf{r}} - \nabla_{\mathbf{r}'})_{\mathbf{r}=\mathbf{r}'} - \frac{e^2 \mathbf{A}(\mathbf{r})}{m}, \tag{29}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential corresponding to an external field which produces an average current given by

$$\mathbf{j}(\mathbf{r}) = \hat{\mathbf{j}}(\mathbf{r}) \int \frac{d\epsilon}{2\pi} G_1^{(K)}(\epsilon, \mathbf{r}, \mathbf{r}')_{\mathbf{r}=\mathbf{r}'}. \tag{30}$$

To calculate the linear response to the vector potential $\mathbf{A}(\mathbf{r})$ it is sufficient to find the Keldysh component of the electron Green's function to the first order:

$$G_1^{(K)}(\epsilon, \mathbf{r}, \mathbf{r}') = G^{(K)}(\epsilon, \mathbf{r}, \mathbf{r}') + \int d\mathbf{r}_1 G^{(R)}(\epsilon, \mathbf{r}, \mathbf{r}_1) e\mathbf{A}(\mathbf{r}_1) \hat{\mathbf{j}}(\mathbf{r}_1) G^{(K)}(\epsilon - \omega_{ext}, \mathbf{r}_1, \mathbf{r}') + \int d\mathbf{r}_1 G^{(K)}(\epsilon, \mathbf{r}, \mathbf{r}_1) e\mathbf{A}(\mathbf{r}_1) \hat{\mathbf{j}}(\mathbf{r}_1) G^{(A)}(\epsilon - \omega_{ext}, \mathbf{r}_1, \mathbf{r}'). \quad (31)$$

We have supposed that the external field oscillates with a finite but small frequency ω_{ext} , so that $A = cE/i\omega_{ext}$. Because we are dealing with a non-superconducting disordered system, the diamagnetic (i.e. the second) term of the current operator on the right hand side of Eq.(29) cancels a ω_{ext}^{-1} contribution to the conductivity from the first term. Note that $\hat{G}(\epsilon, \mathbf{r}, \mathbf{r}')$ is the exact electron Green's function of the electron–environment system, defined by Eq.(7). Since in this section we consider $F_q[\psi^+, \psi] = 1$, we can replace $\hat{G}(\epsilon, \mathbf{r}, \mathbf{r}')$ by $\hat{G}_c(\epsilon, \mathbf{r}, \mathbf{r}')$, defined in Eq.(23).

We shall treat the impurity potential $U(\mathbf{r})$ and electric field $V(t, \mathbf{r})$ defined in Eq.(20) as perturbations, keeping all orders in them. The corresponding electron vertices are proportional to the unit matrix in the Keldysh space. From this observation it follows that there is only one Keldysh component in every conductivity diagram.

We consider here the weak localization correction to conductivity which is given by a maximally crossed diagram, see Fig.1.

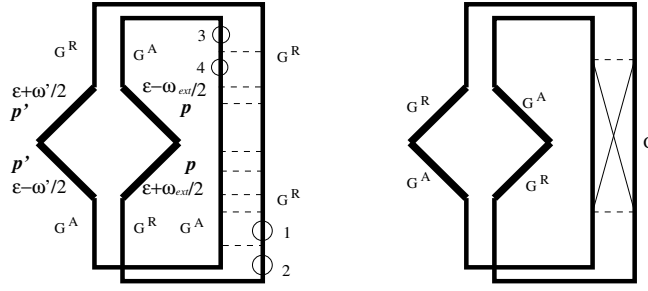


Fig.1a

Fig.1b

FIG. 1. Figure 1a shows maximally crossed diagrams which correspond to the weak localization correction to conductivity. Open circles denote the position for the Keldysh component of the electron Green's function. The sum of all possible diagrams is represented by the diagram in Fig.1b. The Cooperon C satisfies the equation illustrated in Fig.2.

$$C(t_1^+, t_1^+, t_2^+, t_2^+) = \begin{array}{c} G^{(R)} \\ G^{(A)} \\ G^{(R)} \\ G^{(A)} \end{array} \begin{array}{c} t_1^+ \\ t_2^+ \\ t_1^+ \\ t_2^+ \end{array} = \begin{array}{c} G^{(R)} \\ G^{(A)} \\ G^{(R)} \\ G^{(A)} \end{array} \begin{array}{c} t_1^+ \\ t_2^+ \\ t_1^+ \\ t_2^+ \end{array} + \dots$$

$$\Sigma^+(\omega, q) = \begin{array}{c} V(\omega, q) \\ \epsilon_1 p \\ \epsilon_1 + \omega \\ \epsilon_2 - p + k \end{array} \quad \Sigma^-(\omega, q) = \begin{array}{c} \epsilon_1 p + k \\ \epsilon_2 - p \\ \epsilon_2 + \omega \\ V(\omega, q) \end{array}$$

$$C^{(0)}(\omega, k) = \begin{array}{c} \epsilon + \omega \\ \epsilon \\ p + k \\ -p \end{array} = \dots$$

FIG. 2. The Dyson equation for the Cooperon in the classical electric field $V(t, \mathbf{r})$.

The first term in Eq.(31) does not contribute to this correction and two other terms do not vanish only if the Keldysh component stands at the places marked by circles in Fig.1. Otherwise we get integrals

$$\int \frac{d\mathbf{p}}{(2\pi)^d} G^{(A)}(\epsilon, \mathbf{p}) G^{(A)}(\epsilon', \mathbf{p} + \mathbf{k}) = \int \frac{d\mathbf{p}}{(2\pi)^d} G^{(R)}(\epsilon, \mathbf{p}) G^{(R)}(\epsilon', \mathbf{p} + \mathbf{k}) = 0. \quad (32)$$

The Green's functions in Eq.(24) are also impurity renormalized so that

$$G^{(R,A)}(\epsilon, \mathbf{p}) = \frac{1}{\epsilon - \epsilon(p) \pm i/2\tau_{imp}}, \quad (33)$$

and $\epsilon(\mathbf{p}) = \mathbf{p}^2/2m - \mu$ is the energy of a free electron with momentum \mathbf{p} . In equilibrium the Keldysh component satisfies the equation

$$G^{(K)}(\epsilon, \mathbf{p}) = h(\epsilon) \left(G^{(R)}(\epsilon, \mathbf{p}) - G^{(A)}(\epsilon, \mathbf{p}) \right), \quad (34)$$

where $h(\epsilon) = \tanh \epsilon/2T$.

Using Eq.(34) we note that the diagram, which contains the Keldysh component at the position marked by 1 is canceled by a part of the diagram with the Keldysh component at position 2. There is a similar cancellation of some terms of the remaining two diagrams. The result can be represented in the form, shown in Fig.(2). We will keep the leading terms in $\omega\tau_{imp}$ and $D\mathbf{k}^2\tau_{imp}$, where ω and \mathbf{k} are the characteristic energy and momentum changes due to the fluctuating field.

The corresponding analytical expression for the weak localization correction to the conductivity is given by

$$\Delta\sigma_{wl}(\omega') = e^2 \int \frac{d\mathbf{p}d\epsilon}{(2\pi)^{d+1}} \int \frac{d\mathbf{p}'d\epsilon'}{(2\pi)^{d+1}} \mathbf{p} \cdot \mathbf{p}' \mathcal{M}(\epsilon, \epsilon', \mathbf{p}, \mathbf{p}') \frac{(h(\epsilon + \omega_{ext}/2) - h(\epsilon - \omega_{ext}/2))}{\omega_{ext}} \quad (35)$$

$$C(\epsilon' + \omega'/2, \epsilon - \omega_{ext}/2, \epsilon + \omega_{ext}/2, \epsilon' - \omega'/2, \mathbf{p} + \mathbf{p}').$$

Here

$$\mathcal{M}(\epsilon, \epsilon', \mathbf{p}, \mathbf{p}') = G^{(R)}(\epsilon + \omega_{ext}/2, \mathbf{p}) G^{(A)}(\epsilon - \omega_{ext}/2, \mathbf{p}) G^{(R)}(\epsilon' + \omega'/2, \mathbf{p}') G^{(A)}(\epsilon' - \omega'/2, \mathbf{p}') \quad (36)$$

is known as a Hikami box, $C(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \mathbf{p} + \mathbf{p}')$ is the Cooperon, ω' is the current frequency. Since the Cooperon is singular function of $\mathbf{p} + \mathbf{p}'$, we can use the equality $\mathbf{p} \approx -\mathbf{p}'$ everywhere, except as an argument of the Cooperon. Performing the integration over \mathbf{p} and keeping the Cooperon dependence on $\mathbf{p} + \mathbf{p}' = \mathbf{k}$, we get

$$\Delta\sigma_{wl}(\omega') = -4\pi\nu D e^2 \tau_{imp}^2 \int \frac{d\mathbf{k}}{(2\pi)^d} \int \frac{d\epsilon}{2\pi} \frac{(h(\epsilon + \omega_{ext}/2) - h(\epsilon - \omega_{ext}/2))}{\omega_{ext}} \quad (37)$$

$$\int \frac{d\epsilon'}{2\pi} C(\epsilon' + \omega'/2, \epsilon - \omega_{ext}/2, \epsilon + \omega_{ext}/2, \epsilon' - \omega'_{ext}/2, \mathbf{k}).$$

Further calculations are more convenient in the time representation. We define the Fourier transform of the Cooperon by the equation:

$$C(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \mathbf{r}, \mathbf{r}') = \int \int \int \int dt_1^+ dt_1^- dt_2^+ dt_2^- \exp(i(\epsilon_1 t_1^+ + \epsilon_2 t_1^- - \epsilon_3 t_2^+ - \epsilon_4 t_2^-)) \times \quad (38)$$

$$C(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}').$$

Not all four time variables of the Cooperon are independent and we introduce a new notation

$$C(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}') = \mathcal{C}(T, \eta_1, \eta_2, \mathbf{r}, \mathbf{r}') \delta(t_1^+ + t_1^- - t_2^+ - t_2^-), \quad (39)$$

where $2T = t_1^+ + t_1^-$, and $\eta_{1,2} = t_{1,2}^+ - t_{1,2}^-$. For the present purposes one only needs this function for $\eta_1 = -\eta_2 = \eta$. This can be seen by integration over ϵ' in Eq.(37) producing the constraint $\delta(\eta_1 + \eta_2)$, and allowing one time integral to be done. Then one can also complete the integration over ϵ , since the only remaining dependence on ϵ is in the difference of the electron distribution functions $h(\epsilon + \omega_{ext}/2)$ and $h(\epsilon - \omega_{ext}/2)$. After these steps one reaches the formula

$$\Delta\sigma_{wl}(\omega_{ext} = 0) = -\frac{4e^2 D}{\pi} \int_0^\infty d\eta P(\eta), \quad (40)$$

where $P(\eta)$, which has the meaning of the probability of returning to a starting point after a time η , is the Cooperon evaluated at coinciding space and time points,

$$P(\eta) = 2\pi\nu\tau_{imp}^2 \mathcal{C}(T, +\eta, -\eta, \mathbf{r}, \mathbf{r}) \quad (41)$$

To evaluate the Cooperon we can use the following path integral form (see Appendix A for more details).

$$\mathcal{C}(T, \eta_1, \eta_2, \mathbf{r}, \mathbf{r}') = \frac{1}{2\pi\nu\tau_{imp}^2} \int_{r(\eta_2)=\mathbf{r}'}^{r(\eta_1)=\mathbf{r}} \mathcal{D}\mathbf{r}(t) \exp\left(-\int_{\eta_2}^{\eta_1} d\zeta \left(\frac{\dot{\mathbf{r}}^2(\zeta)}{4D} - i\tilde{V}(T, \zeta, \mathbf{r}(\zeta))\right)\right), \quad (42)$$

with

$$\tilde{V}(T, \zeta, \mathbf{r}(\zeta)) = V(T + \zeta/2, \mathbf{r}(\zeta)) - V(T - \zeta/2, \mathbf{r}(\zeta)), \quad (43)$$

and the electric field potential $V(t, \mathbf{r})$ defined by Eq.(20). In performing the integration over the magnitude and the phase of the electric field according to Eq.(18), we note that the time variable T can be absorbed into the phase ϕ_ν of the field. Consequently, the correction to the conductivity is independent of T . As the result of the integration we get for a wire, in which case \mathbf{r} is one dimensional,

$$\mathcal{C}(+\eta, -\eta, r, r') = \int_{r(-\eta)=r'}^{r(+\eta)=r} \mathcal{D}r(t) \exp\left(-\int_{-\eta}^{+\eta} dt \left(\frac{\dot{r}^2(t)}{4D} + U(r, t)\right)\right), \quad (44)$$

where

$$U(r, t) = \begin{cases} \frac{e^2 T R_{eff}}{L^2} (r(t) - r(-t))^2, & \text{for the Nyquist noise model,} \\ \frac{2e^2 T}{\sigma_1} |r(t) - r(-t)|, & \text{for the electron-electron interaction.} \end{cases} \quad (45)$$

Here we have used the specific time variables $\eta_1 = \eta$ and $\eta_2 = -\eta$, needed for the weak localization correction to the conductivity.

At this point it is convenient to introduce new variables for the path coordinates. Let us define $R(t) = (r(t) + r(-t))/\sqrt{2}$ and $x(t) = (r(t) - r(-t))/\sqrt{2}$. Then we can eliminate the integration over negative time. Also this change of variables separates $R(t)$ and $x(t)$ in the exponent. The motion described by $R(t)$ is an ordinary diffusion and a direct integration gives unity for the whole integral. The path integral over $x(t)$ can be done easily for the Nyquist noise model, because the motion corresponding to $x(t)$ is that of a particle in a harmonic oscillator which is at the position of minimum potential energy at the starting and final points. Thus, Nyquist noise yields the following simple explicit result for the Cooperon

$$\mathcal{C}(\eta, -\eta, \mathbf{r}, \mathbf{r}') = \frac{\sqrt{2}}{4\pi\nu\tau_{imp}^2} \int_0^{x(\eta)=(\mathbf{r}'-\mathbf{r})/\sqrt{2}} \mathcal{D}x(t) \exp\left(-\frac{1}{4D} \int_0^\eta dt (\dot{x}^2(t) + \Omega^2 x^2(t))\right) = \frac{1}{2\pi\nu\tau_{imp}^2} \sqrt{\frac{\Omega}{8\pi D \sinh \Omega \eta}} \exp\left(-\frac{\Omega(r' - r)^2}{8D} \coth \Omega \eta\right), \quad (46)$$

where

$$\Omega^2 = 16 \frac{e^2 D T R_{eff}}{L^2} = 16 T T_q \quad (47)$$

and $T_q = D T e^2 R_{eff} / L^2$.

We thus have

$$P(\eta) = \sqrt{\frac{\Omega}{8\pi D \sinh \Omega \eta}}, \quad (48)$$

from which it follows that in this model

$$\Delta\sigma_{wl} = -\frac{\sqrt{2}e^2}{\sqrt{\pi^3}} I_0 \sqrt{\frac{D}{\Omega}}, \quad (49)$$

where

$$I_0 = \int_0^\infty \frac{dx}{\sqrt{\sinh x}} \approx 3.71. \quad (50)$$

So far we have assumed that the interaction with the environment is the only source of the dephasing. If the sample is in a magnetic field H we have to consider the competition between the dephasing produced by the interaction and

the magnetic field. The magnetic field exponentially suppresses the Cooperon (Eq.(42)) as a function of $\eta_2 - \eta_1$. The magnetic characteristic time is $\tau_H^{-1} = e^2 D H^2 a^2 / 3c^2$, where a is the thickness of the wire, see [1]. When both sources of dephasing are present the weak localization correction to conductivity is given by

$$\Delta\sigma_{wl} = -\frac{4e^2 D}{\pi} \int_0^\infty d\eta P(\eta) e^{-2\eta/\tau_H}. \quad (51)$$

This formula allows us to reproduce in a different way from theirs some results of [3] for $\Delta\sigma_{wl}$ in two limits. For $\tau_H \Omega \ll 1$ we can expand $(\sinh \Omega \eta)^{-1/2} = 1/\sqrt{\Omega \eta} - (\Omega \eta)^{3/2}/12$ and obtain

$$\Delta\sigma_{wl} = -\frac{e^2}{\pi} \sqrt{D \tau_H} \left(1 - \frac{1}{64} (\tau_H \Omega)^2 \right). \quad (52)$$

In the opposite limit, $\tau_H \Omega \gg 1$ we expand $\exp(-\eta/\tau_H) \approx 1 - \eta/\tau_H$ and integrate over η to get

$$\Delta\sigma_{wl} = -\frac{\sqrt{2}e^2}{\pi^{3/2}} \sqrt{\frac{D}{\Omega}} \left(I_0 - I_1 \frac{2}{\tau_H \Omega} \right), \quad (53)$$

where

$$I_1 = \int_0^\infty \frac{\zeta}{\sqrt{\sinh \zeta}} d\zeta \approx 5.84. \quad (54)$$

In the case of the electron–electron interaction, the functional integral has been related to a Schrödinger-like differential equation in ref. [3] with the result

$$\Delta\sigma_{wl}^{ee} = \frac{e^2}{\pi} \sqrt{D \gamma_{ee}^{-1}} \frac{1}{(\ln \text{Ai}(1/\tau_H \gamma))^7}, \quad (55)$$

where $\text{Ai}(x)$ is the Airy function and

$$\gamma_{ee} = \left(\frac{e^2 T \sqrt{D}}{\sigma_1} \right)^{2/3}. \quad (56)$$

$L_{ee} = \sqrt{D/\gamma_{ee}}$ and $L_H = \sqrt{D \tau_H}$.

In weak magnetic field $L_H \gg L_{ee}$

$$\Delta\sigma_{wl}^{ee} = -\frac{2e^2 L_{ee}}{3^{5/6} \Gamma^2(2/3)}. \quad (57)$$

where $L_{ee} = \sqrt{D/\gamma_{ee}}$ and $L_H = \sqrt{D \tau_H}$. In the opposite case we use

$$\text{Ai}(x) \sim \frac{1}{2(\pi^2 x)^{1/4}} e^{-2/3 x^{3/2}} \quad (58)$$

to get

$$\Delta\sigma_{wl}^{ee} = -\frac{e^2 L_H}{\pi} \left(1 - \frac{1}{4} (\tau_H \gamma)^{3/2} \right). \quad (59)$$

Note, that the result of Eq.(56) differs from one found in [3] by a factor of 2. On the other hand the expansion Eq.(59) is consistent with the result of [5]. Also the result of Eq.(49) has an extra $2^{-1/4}$ to the numerical factor $\Gamma(1/4)/2\pi\Gamma(3/4)$ found in [3] for the weak localization correction to conductivity in the presence of the Nyquist noise.

We have thus seen that our 'classical' Action exactly reproduces known high temperature results for the intrinsic dephasing effect in weak localization without the need for an external infrared cut-off. In the remainder of this paper we build on these calculations to examine quantum corrections.

IV. QUANTUM CORRECTIONS

Now we consider the contribution of the quantum part of the influence functional to the weak localization correction to conductivity. In this section we consider dephasing correction to conductivity due to the Nyquist noise. In this case the calculations can be done analytically. The calculations described here are similar to those in ref. [5]. The difference is that ours are free from infrared divergences, because the low frequency modes of interaction have already been taken into account exactly.

Expanding $F_q[\psi^+, \psi]$, the second term in Eq.(16), to the first order we obtain

$$F_q[\psi^+, \psi] \approx 1 - \frac{1}{2} \int_{-\infty}^{+\infty} ds \int_{-\infty}^{+\infty} du \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\mathcal{L}^{(K)}(s-u, \mathbf{r}_1, \mathbf{r}_2)(q(s, \mathbf{r}_1) - q'(s, \mathbf{r}_1))(q(u, \mathbf{r}_2) - q'(u, \mathbf{r}_2)) + \mathcal{L}^{(R)}(s-u, \mathbf{r}_1, \mathbf{r}_2)(q(s, \mathbf{r}_1) - q'(s, \mathbf{r}_1))(q(u, \mathbf{r}_2) + q'(u, \mathbf{r}_2)) + \mathcal{L}^{(A)}(s-u, \mathbf{r}_1, \mathbf{r}_2)(q(u, \mathbf{r}_1) - q'(u, \mathbf{r}_1))(q(s, \mathbf{r}_2) + q'(s, \mathbf{r}_2)) \right), \quad (60)$$

where Keldysh labels have been assigned according to the definitions

$$\begin{aligned} \mathcal{L}^{(R)}(t, \mathbf{r}_1, \mathbf{r}_2) &= \Theta(t) \sum_{\nu} e^2 \frac{\phi_{\nu}(\mathbf{r}_1) \phi_{\nu}(\mathbf{r}_2)}{4M_{\nu}\omega_{\nu}} \exp(-i\omega_{\nu}t), \\ \mathcal{L}^{(A)}(t, \mathbf{r}_1, \mathbf{r}_2) &= -\Theta(-t) \sum_{\nu} e^2 \frac{\phi_{\nu}(\mathbf{r}_1) \phi_{\nu}(\mathbf{r}_2)}{4M_{\nu}\omega_{\nu}} \exp(-i\omega_{\nu}t), \\ \mathcal{L}^{(K)}(t, \mathbf{r}_1, \mathbf{r}_2) &= \sum_{\nu} e^2 \frac{\phi_{\nu}(\mathbf{r}_1) \phi_{\nu}(\mathbf{r}_2)}{4M_{\nu}\omega_{\nu}} \left(\coth \frac{\omega_{\nu}}{2T} - \frac{2T}{\omega_{\nu}} \right) \cos(\omega_{\nu}t). \end{aligned} \quad (61)$$

As we have already mentioned in the previous section, the conductivity is determined by the Keldysh component of the electron Green's function (see Eq.(30)). The Keldysh component can be represented in terms of the Green's function defined in Eq.(7):

$$G^{(K)}(t, t', \mathbf{r}, \mathbf{r}') = G(t_f, t'_b, \mathbf{r}, \mathbf{r}') + G(t_b, t'_f, \mathbf{r}, \mathbf{r}') \quad (62)$$

where the f -index of time argument means that it is taken at the forward part of the Keldysh contour, and the b -index for the backward part, so that $t_f <_K t_b$. The average in Eq.(7) is represented by Eq.(22) with respect to the classical part of the effective action $F_c[\psi^+, \psi]$, see Eq.(18) and the quantum part in the form of Eq.(61). The first term in Eq.(61), which is unity, provides the expression for $G^{(K)}(t, t', \mathbf{r}, \mathbf{r}')$ considered in the previous section and the remaining part of Eq.(61) gives the first order quantum correction. Using the Wick's theorem for the electron operators we get after cumbersome calculations the quantum correction to the Keldysh component of the Green's function

$$\begin{aligned} G_{1 \times int}^{(K)}(\mathbf{x}, \mathbf{x}') &= \int d\mathbf{x}_1 \int d\mathbf{x}_2 \left(\mathcal{L}^{(K)}(\mathbf{x}_1, \mathbf{x}_2) \left(G_1^{(R)}(\mathbf{x}, \mathbf{x}_1) G_1^{(R)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(K)}(\mathbf{x}_2, \mathbf{x}') + \right. \right. \\ &G_1^{(R)}(\mathbf{x}, \mathbf{x}_1) G_1^{(K)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(A)}(\mathbf{x}_2, \mathbf{x}') + G_1^{(K)}(\mathbf{x}, \mathbf{x}_1) G_1^{(A)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(A)}(\mathbf{x}_2, \mathbf{x}') \left. \right) + \\ &\mathcal{L}^{(R)}(\mathbf{x}_1, \mathbf{x}_2) \left(G_1^{(R)}(\mathbf{x}, \mathbf{x}_1) G_1^{(K)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(K)}(\mathbf{x}_2, \mathbf{x}') + G_1^{(R)}(\mathbf{x}, \mathbf{x}_1) G_1^{(R)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(A)}(\mathbf{x}_2, \mathbf{x}') + \right. \\ &\left. \mathcal{L}^{(A)}(\mathbf{x}_1, \mathbf{x}_2) \left(G_1^{(K)}(\mathbf{x}, \mathbf{x}_1) G_1^{(K)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(A)}(\mathbf{x}_2, \mathbf{x}') + G_1^{(R)}(\mathbf{x}, \mathbf{x}_1) G_1^{(A)}(\mathbf{x}_1, \mathbf{x}_2) G_1^{(A)}(\mathbf{x}_2, \mathbf{x}') \right) \right), \end{aligned} \quad (63)$$

where $\mathbf{x} = (t, \mathbf{r})$. Subscript $_1$ means that the Green's functions should be calculated up to the first order in the external electric field $\mathbf{A}(\mathbf{r})$, similarly to what has been done in Eq.(30). Thus, the Keldysh component is given by Eq.(31) and $G_1^{(R,A)}(t, t', \mathbf{r}, \mathbf{r}')$ are

$$\begin{aligned} G_1^{(R)}(t, t', \mathbf{r}, \mathbf{r}') &= G_c^{(R)}(t, t', \mathbf{r}, \mathbf{r}') + \int d\mathbf{r}_1 \int dt_1 G_c^{(R)}(t, t', \mathbf{r}, \mathbf{r}_1) e\mathbf{A}(t_1, \mathbf{r}_1) \mathbf{j}(\mathbf{r}_1) G_c^{(R)}(t_1, t', \mathbf{r}_1, \mathbf{r}'), \\ G_1^{(A)}(t, t', \mathbf{r}, \mathbf{r}') &= G_c^{(A)}(t, t', \mathbf{r}, \mathbf{r}') + \int d\mathbf{r}_1 \int dt_1 G_c^{(A)}(t, t', \mathbf{r}, \mathbf{r}_1) e\mathbf{A}(t_1, \mathbf{r}_1) \mathbf{j}(\mathbf{r}_1) G_c^{(A)}(t_1, t', \mathbf{r}_1, \mathbf{r}') \end{aligned} \quad (64)$$

Eq.(63) is in agreement with the expression for the first order correction to the Keldysh component of the electron Green's function obtained in the standard technique. [See [10] for a general discussion and [5] for the explicit form.]

It was shown in paper [5] that to second order in the interaction the weak localization correction to the conductivity can be represented as a sum of two terms, called the dephasing term and the cross term. The dephasing term contains the contribution from the electron interaction with Boson modes having energies smaller than temperature of the system. On the other hand the cross term has contribution from the whole spectrum, and corresponds to the electron scattering from the Friedel oscillations. According to [5] [c.f. Eq.(1) above,] the interaction correction term has a weaker divergence, $\Delta\sigma_{cr} \sim \sqrt{\tau_\varphi}$ (where τ_φ is the decoherence time), than the dephasing term $\Delta\sigma_{deph} \sim \tau_\varphi^2$. Keeping this in mind, we restrict our calculations to this latter term, which is expected to give the most important correction to the semiclassical calculations discussed in the preceding section.

The derivation of the dephasing term of the weak localization correction is described in Appendix B. The corresponding diagrams are shown in Fig.3. In general the result may be represented as a product of three Cooperons, see Eqs.(B1),(B4). The whole process corresponds to the diffusion of an electron from an initial point to some other point, where it emits a boson. Then it travels to the second point and absorbs the same boson, after that it goes back to the initial point.

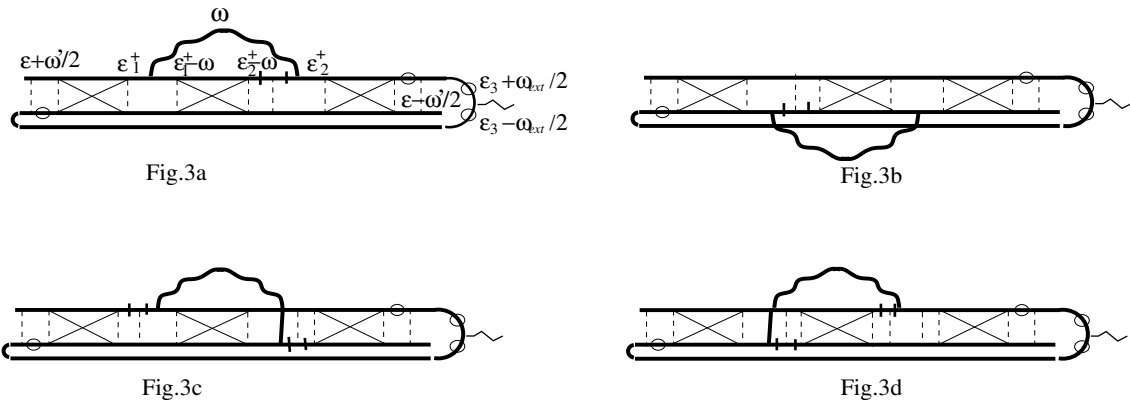


FIG. 3. Four possible diagrams which contribute to the dephasing correction to the conductivity.

In the case when the propagation of the emitted boson is described by the Keldysh component of the boson Green's function, the Cooperons have the same time arguments at intermediate points so that the diffusive motion is continuous in time, see Eq.(B1). Terms which have the retarded or advanced components of the boson Green's function have a discontinuity at the points which represent the nondiagonal vertices in the Keldysh space, Eq.(B4). This discontinuity can be explained by the fact that the energy of an electron is not conserved along its trajectory because of the interaction with the classical field. We disregard these discontinuities and set $\tau = 0$ in the Cooperons of Eq.(B4). Then the integrals over τ are trivial. It is possible to show that in this case we disregard terms which are of the same order as higher order quantum corrections.

The dephasing correction to conductivity is

$$\Delta\sigma_{deph} = \frac{4De^2}{\pi} \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} \frac{2T}{\omega} f(\omega/2T) J(\omega, q) \int_0^{+\infty} d\eta \mathcal{C}_2(T, \eta, \mathbf{r}), \quad (65)$$

where

$$\mathcal{C}_2(T, \eta, \mathbf{r}) = 2 \int_{-\eta}^{+\eta} d\zeta_1 \int_{-\eta}^{\zeta_1} d\zeta_2 \int d\mathbf{r}_1 \int d\mathbf{r}_2 (\cos \omega(\zeta_1 - \zeta_2) - \cos \omega(\zeta_1 + \zeta_2)) e^{-iq(\mathbf{r}_1 - \mathbf{r}_2)} \times \mathcal{C}(T, \eta, \zeta_1, \mathbf{r}, \mathbf{r}_1) \mathcal{C}(T, \zeta_1, \zeta_2, \mathbf{r}_1, \mathbf{r}_2) \mathcal{C}(T, \zeta_2, \eta, \mathbf{r}_2, \mathbf{r}). \quad (66)$$

with the thermal function

$$f(x) = \frac{x^2}{\sinh^2 x} - 1. \quad (67)$$

Note that $f(x)$ vanishes at $x \approx 0$, i.e in the infrared region, because of the inclusion of classical fluctuations in the factors \mathcal{C} .

We can eliminate the integrals over $\mathbf{r}_{1,2}$ and represent $\mathcal{C}_2(T, \eta, \mathbf{r})$ as

$$\mathcal{C}_2(T, \eta, \mathbf{r}) = 2 \int_{-\eta}^{+\eta} d\zeta_1 \int_{-\eta}^{\zeta_1} d\zeta_2 (\cos \omega(\zeta_1 - \zeta_2) - \cos \omega(\zeta_1 + \zeta_2)) \exp(-i\mathbf{q}(\mathbf{r}(\zeta_1) - \mathbf{r}(\zeta_2))) \mathcal{C}(T, +\eta, -\eta, \mathbf{r}, \mathbf{r}), \quad (68)$$

and $\mathcal{C}(T, +\eta, -\eta, \mathbf{r}, \mathbf{r})$ is given by Eq.(42) without factor $(2\pi\nu\tau_{imp}^2)^{-1}$.

Now we can easily perform averaging over the fluctuations of the classical field according to Eq.(18). As a result we find, that now we can substitute the Cooperon defined by Eq.(44). The path integral can be simplified by using the variables $R(t)$ and $x(t)$ introduced below Eq.(43). Since the integrals over $R(t)$ and $x(t)$ are defined only at positive values of t , we have to rewrite Eq.(68) as an integral over positive values of ζ_1 and ζ_2 .

In terms of the R and x variables the averaged $\mathcal{C}_2(T, \eta, \mathbf{r})$ has the form

$$\begin{aligned} \mathcal{C}_2(T, \eta, \mathbf{r}) &= 4 \int_0^{+\eta} d\zeta_1 \int_0^{+\zeta_1} d\zeta_2 (\cos \omega(\zeta_1 - \zeta_2) - \cos \omega(\zeta_1 + \zeta_2)) \\ &\frac{1}{\sqrt{2}} \int_{-\infty}^{+\infty} dR_1 \int_{R(0)=R_1}^{R(\eta)=r/\sqrt{2}} \mathcal{D}R(t) \exp\left(-\int_0^\eta dt \frac{\dot{R}^2(t)}{4D}\right) e^{-iq(R(\zeta_1)-R(\zeta_2))/\sqrt{2}} \\ &\int_{x(0)=0}^{x(\eta)=0} \mathcal{D}x(t) \exp\left(-\int_0^\eta dt \frac{\dot{x}^2(t) + \Omega^2 x^2(t)}{4D}\right) e^{iqx(\zeta_2)/\sqrt{2}} (-2i) \sin \frac{q}{\sqrt{2}} x(\zeta_1). \end{aligned} \quad (69)$$

The integral over $R(t)$, written in the second line of Eq.(69), can be done:

$$\int_{-\infty}^{+\infty} dR_1 \int_{R(0)=R_1}^{R(\eta)=r/\sqrt{2}} \mathcal{D}R(t) \exp\left(-\int_0^\eta dt \frac{\dot{R}^2(t)}{4D}\right) e^{-iq(R(\zeta_1)-R(\zeta_2))/\sqrt{2}} = e^{-Dq^2(\zeta_1-\zeta_2)/2}. \quad (70)$$

The integral over the odd part of trajectories is more complicated. We will represent the third line in the form:

$$\begin{aligned} &\int_{x(0)=0}^{x(\eta)=0} \mathcal{D}x(t) \exp\left(-\int_0^\eta dt \frac{\dot{x}^2(t) + \Omega^2 x^2(t)}{4D}\right) e^{iqx(\zeta_2)/\sqrt{2}} \sin \frac{q}{\sqrt{2}} x(\zeta_1) = \\ &\int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dx_2 \tilde{\mathcal{C}}(0, x_2, \zeta_2) \tilde{\mathcal{C}}(x_2, x_1, \zeta_1 - \zeta_2) \tilde{\mathcal{C}}(x_1, 0, \eta - \zeta_1) e^{iqx_2/\sqrt{2}} \sin \frac{q}{\sqrt{2}} x_1, \end{aligned} \quad (71)$$

where

$$\begin{aligned} \tilde{\mathcal{C}}(x_1, x_2, \zeta) &= \int_{x(0)=x_2}^{x(\zeta)=x_1} \mathcal{D}x(t) \exp\left(-\int_0^\zeta dt \frac{\dot{x}^2(t) + \Omega^2 x^2(t)}{4D}\right) = \\ &\sqrt{\frac{\Omega}{4\pi D \sinh \Omega \zeta}} \exp\left(-\frac{\Omega}{4D \sinh \Omega \zeta} ((x_1^2 + x_2^2) \cosh \Omega \zeta - 2x_1 x_2)\right) \end{aligned} \quad (72)$$

is the Green's function of a harmonic oscillator in imaginary time.

The integrals over x_1 and x_2 are Gaussian and can be done exactly for arbitrary value of q . In our case q is bounded by $q_c \ll 1/L$. It allows us to perform the expansion in powers of q . Keeping the first non-vanishing term we get

$$\Delta\sigma_{deph} = \frac{-\Delta\sigma_{wl}\Omega^2}{I_0} \int \frac{d\omega}{2\pi} f(\omega/2T) \int_0^{+\infty} d\eta I(\eta, \omega), \quad (73)$$

where

$$\begin{aligned} I(\eta) &= \int_0^{+\eta} d\zeta_1 \int_0^{+\zeta_1} d\zeta_2 (\cos \omega(\zeta_1 - \zeta_2) - \cos \omega(\zeta_1 + \zeta_2)) \frac{\sinh \Omega \zeta_2 \sinh \Omega(\eta - \zeta_1)}{(\sinh \Omega \eta)^{3/2}} = \\ &\frac{1}{\Omega^2 + \omega^2} \left(\frac{\Omega \eta}{2\sqrt{\sinh \Omega \eta}} + \frac{\Omega(\omega^2 - \Omega^2)}{4\omega(\omega^2 + \Omega^2)} \frac{\sin 2\omega \eta}{\sqrt{\sinh \Omega \eta}} - \frac{\Omega^2}{\omega^2 + \Omega^2} \frac{\sin^2 \omega \eta \cosh \Omega \eta}{\sqrt{\sinh^3 \Omega \eta}} \right). \end{aligned} \quad (74)$$

Now we can perform integral over frequency. The second and third terms in Eq.(74) converge fast and we can use an approximation for the thermal function at small values: $f(x) = -x^2/3$. The first term can be evaluated in the lowest order in Ω/T . As the result we get

$$\Delta\sigma_{deph} = \Delta\sigma_{wl} \left(1 + \frac{I_1}{4} \frac{\Omega}{T} + \frac{2I_2 - I_3}{96} \frac{\Omega^2}{T^2} \right), \quad (75)$$

where I_1 was defined by Eq.(54),

$$I_2 = \int_0^\infty \frac{(1-2z)e^{-z}}{\sqrt{\sinh z}} dz \approx 1.02, \quad (76)$$

and

$$I_3 = \int_0^\infty \frac{(2z-1)e^{-2z}+1}{\sqrt{\sinh z}} \coth z dz \approx 7.54. \quad (77)$$

The third term in the brackets is of second order in Ω/T and can be omitted.

We see that the quantum corrections are not important, unless $T \leq T_q$, with T_q being defined by Eq.(47). Let us evaluate dephasing rate $\Omega(T_q)$ at $T = T_q$. According to Eq.(47) we have

$$\Omega(T_q) = \frac{4DR_{eff}}{L^2}. \quad (78)$$

Here we use dimensionless resistance in terms of \hbar/e^2 . We substitute $\Omega(T_q)$ to Eq.(49), see also Eq.(28), and get

$$\Delta\sigma_{wl} \sim -\sigma \sqrt{\frac{(R_w + R_0)^2}{R_0}}. \quad (79)$$

The weak localization becomes strong localization at $T \gg T_q$, if $(R_w + R_0)^2/R_0 \gg 1$. In the opposite case $(R_w + R_0)^2/R_0 \ll 1$ we have $R_{eff} \ll 1$, consequently $\Omega(T_q) \ll D/L^2$ and the wire becomes zero-dimensional at high temperature. We conclude that in both limits the weak localization correction to conductivity deviates from the result of [3] (see Eq.(40) at higher temperature than T_q due to other reasons rather than due to the quantum corrections.

The plus sign before the second term in Eq.(75) means that the dephasing rate found in [3] is overestimated, and the quantum correction suppresses it.

V. ELECTRON-ELECTRON INTERACTION

Now we discuss how the result obtained in the previous section can be generalized to the case of the real electron-electron interaction. The screened Coulomb spectral function of a dirty metal is given by Eq.(26), where σ_1 is the one dimensional conductivity of the wire and the momentum integral in Eq.(25) runs from $-\infty$ to $+\infty$. [13] We will use the effective action in the form of Eq.(10) with the appropriate choice of mode density Eq.(26) to make a semi-quantitative calculation of the quantum correction to the semiclassical result Eq.(55).

First consider weak localization neglecting the quantum correction. This problem was solved in [3]. [See also section 3 in the present paper.] We discuss how this result can be obtained by qualitative arguments.

The weak localization correction to conductivity is still given by Eq.(37) and the Cooperon can be represented in the form of Eq.(42). Although Eq.(42) was derived for small momentum \mathbf{q} of the electric field $V(\omega, \mathbf{q})$ —see Appendix A—we can use it for the Coulomb interaction with unbounded spectrum in the momentum space, since the main contribution comes from the long wavelength part of the interaction. In higher dimensions $D = 2, 3$, the momentum integration has to be cut to satisfy conditions at which the Cooperon has been derived.

The next step is to average over the fluctuations of the classical field. The Cooperon is defined by Eq.(44) with the ‘potential’ $U(\mathbf{r})$ given by the second line in Eq.(45). Now the exponent is not an analytic function of the coordinate variables $x(t)$.

We can rewrite the exponent of Eq.(44) in terms of dimensionless variables $y = r/L_{ee}$ (space) and $\zeta = t\gamma^{-1}$ (time). Then L_{ee} and γ^{-1} determine the characteristic space and time scales at which the electron-electron interaction becomes important. At smaller scales the interaction is not important and the diffusion can be considered without interaction. In the opposite case the diffusion process is suppressed by the interaction. According to Eq.(44) and Eq.(45) L_{ee} and γ^{-1} are given by

$$\gamma = \frac{D}{L_{ee}^2} = \left(\frac{e^2 T \sqrt{D}}{\sigma_1} \right)^{2/3}. \quad (80)$$

This result was found for the first time in [3].

This intrinsic cutoff may be introduced into the expression for the weak localization correction for noninteracting electrons in the time representation, by cutting off the time integral with the exponential function $\exp(-2t\gamma)$.

$$\Delta\sigma_{wl}^{ee} \approx -\frac{4e^2D}{\pi} \int_0^\infty e^{-2\gamma\tau} \frac{d\tau}{\sqrt{8\pi D\tau}} = \frac{e^2}{\pi} \sqrt{\frac{D}{\gamma}}. \quad (81)$$

This expression is in agreement with the exact result found in [3]. [See Eq.(57).]

The analysis of Appendix B can be directly applied to the case of electron–electron interaction with unbounded momentum integration. It follows that for the dephasing term of the weak localization correction to the conductivity we get Eq.(65) with $\mathcal{C}_2(T, \eta, \mathbf{r})$ given by Eq.(66), which again can be represented in the form of Eq.(68). Averaging over the fluctuations of the electric field $V(t, \mathbf{r})$ gives an expression for $\mathcal{C}_2(T, \eta, \mathbf{r})$, similar to that in Eq.(69):

$$\begin{aligned} \mathcal{C}_2(T, \eta, \mathbf{r}) = & \int_0^{+\eta} d\zeta_1 \int_0^{+\zeta_1} d\zeta_2 (\cos \omega(\zeta_1 - \zeta_2) - \cos \omega(\zeta_1 + \zeta_2)) \\ & \int_{-\infty}^{+\infty} dR_1 \int_{R(0)=R_1}^{R(\eta)=r/\sqrt{2}} \mathcal{D}R(t) \exp\left(-\int_0^\eta dt \frac{\dot{R}^2(t)}{4D}\right) e^{-iq(R(\zeta_1)-R(\zeta_2))} \\ & \int_{x(0)=0}^{x(\eta)=0} \mathcal{D}x(t) \exp\left(-\int_0^\eta dt \left(\frac{\dot{x}^2(t)}{4D} + e^2 \frac{\sqrt{2}}{\sigma_1} T|x(t)|\right)\right) e^{iqx(\zeta_2)} (-2i) \sin \frac{q}{\sqrt{2}} x(\zeta_1). \end{aligned} \quad (82)$$

We have changed variables for the path integral, introducing even and odd parts, as discussed in section 2. The integral over the even part of trajectories is not disturbed by the interaction and the result of integration is given by Eq.(70). The path integral over $x(t)$ can be rewritten in the form of Eq.(71), where $\tilde{\mathcal{C}}(t, x_1, x_2)$ is a solution to:

$$\left(\frac{\partial}{\partial t} - D\nabla_{x_1}^2 + \frac{\sqrt{2}e^2}{\sigma_1} T|x_1|\right) \tilde{\mathcal{C}}(t, x_1, x_2) = \delta(t)\delta(x_1 - x_2). \quad (83)$$

This equation can be rewritten in the dimensionless variables, if time and space coordinates are divided by γ^{-1} and L_{ee} . After this the solution can be found numerically.

The solution to Eq.(83) has the following important property: at small time $t \ll \gamma^{-1}$ it is very similar to the Cooperon without interaction, but at time scale greater than T_{ee} it is suppressed. To evaluate the right hand side of Eq.(73), we can use the Cooperon without interaction, see Eq.(A3), but introduce the upper cutoff for time integrals at γ^{-1} . More exactly, we introduce exponent weight factor, which vanishes at time greater than T_{ee} , and substitute $\tilde{\mathcal{C}}(t, x_1, x_2)$ by the expression given by

$$C(\omega, k) = \frac{1}{i\omega + Dk^2 + \gamma}. \quad (84)$$

In this case calculations can be easily completed. The dephasing part of the weak localization correction to conductivity is given by an analogue to Eq.(73):

$$\Delta\sigma_{deph}^{ee} = \frac{8De^4T}{\pi\sigma_1} \int \frac{d\omega}{2\pi} f(\omega/2T) \int_0^\infty d\eta I(\eta, \omega), \quad (85)$$

where

$$I(\eta, \omega) = \int_0^{+\eta} dx \int_{-\eta+x}^{+\eta-x} dy (\cos \omega x - \cos \omega y) \frac{\sqrt{x(\eta-x)}}{\eta} \exp(-2\eta\gamma). \quad (86)$$

The factor $e^{-\eta\gamma}$ produces the upper cutoff of the time integral. An analytical expression for $\int_0^\infty I(\eta) d\eta$ can be found. This expression is cumbersome and we present only a term which has the weakest frequency dependence. We would like to remind, that now we consider high frequency contribution $\omega \sim T$, since the low frequency part is taken into account in terms of the classical field.

$$\int_0^\infty I(\eta, \omega) d\eta = -\frac{\pi}{2} \frac{\cos\left(\frac{3}{2} \arctan \frac{\omega}{\gamma}\right)}{\sqrt{\gamma^3(\omega^2 + \gamma^2)^{3/4}}} + \dots \sim \frac{\pi}{(2\gamma\omega)^{3/2}}. \quad (87)$$

Substituting this expression into Eq.(73), we find an analogue of Eq.(75):

$$\Delta\sigma_{deph}^{ee} = J \frac{e^2}{\pi} \sqrt{\frac{D}{T}} \sim \Delta\sigma_{wl}^{ee} \sqrt{\frac{\gamma}{T}}, \quad (88)$$

where

$$J = 2 \int dx \frac{1}{x^{3/2}} \left(\frac{x^2}{\sinh^2 x} - 1 \right) \approx -3.66. \quad (89)$$

Let us introduce the dimensionless conductance of the wire at length L :

$$g(L) = \frac{\hbar}{e^2} \frac{\sigma_1}{L}, \quad (90)$$

where σ_1 is the Drude (bare) conductivity of the wire. Then Eq.(80) can be rewritten in the form:

$$\gamma = \frac{T}{g(L_{ee})}. \quad (91)$$

Since in the weak localization regime $g(L_{ee}) \gg 1$, Eqs.(88) and (90) mean that the quantum correction to the classical result obtained in [3] is proportional to a small quantity $[g(L_{ee})]^{-1/2}$. At sufficiently low temperature $g(L_{ee})$ approaches unity, but in this case the classical correction to the conductivity becomes comparable with the Drude conductivity σ_1 , and the perturbation treatment of localization breaks down. The conclusions of this section and the last are thus the same: in the region where the correction to the conductivity may be described by the interference of two time reversed trajectories, neglecting more complicated interference terms, the quantum fluctuations of the dissipative environment are unimportant.

VI. DISCUSSION

In this paper, we have constructed a bridge between semiclassical calculations [3] which keep the interaction to all orders of perturbation theory and exact quantum mechanical calculations to first order in the interaction. [5] While the semiclassical calculations are self consistent and do not require some external cutoff, the finite order perturbation theory is infrared divergent, and requires a low frequency cutoff. By contrast, our calculation of quantum corrections is intrinsically regularized at low frequencies and the parameter of the perturbation theory is γ/T , see Eq.(88).

These differences notwithstanding, we shall now show that when the cutoff introduced in [5] is treated as a parameter to be determined self consistently, one obtains agreement with our results and conclusions as presented in the last section.

The weak localization correction to conductivity found in [5], Eq.(4.13a):

$$\Delta\sigma_{wl} = \Delta\sigma_{wl}^{(0)} + \Delta\sigma_{deph} + \Delta\sigma'_{deph} + \Delta\sigma_{cwl}. \quad (92)$$

The first term $\Delta\sigma_{wl}^{(0)}$ comes from the maximally crossed diagram without electron-electron interaction with finite dephasing rate γ . The result for $\Delta\sigma_{wl}^{(0)}$ is:

$$\Delta\sigma_{wl}^{(0)} = -\frac{2De^2}{\pi} \int \frac{dk}{2\pi} C(\omega = 0, k) = -\frac{e^2}{\pi} \sqrt{D\gamma^{-1}}, \quad (93)$$

where $C(\omega, k)$ is given by Eq.(84).

The first order correction due to the interaction is given by an equation similar to Eq.(65), see [5]

$$\Delta\sigma_{deph} = \frac{4De^2}{\pi} \int \frac{dk}{2\pi} \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} \frac{2T}{\omega} F(\omega/2T) J(\omega, q) C_2(\omega, \mathbf{k}, \mathbf{q}), \quad (94)$$

where

$$C_2(\omega, \mathbf{k}, \mathbf{q}) = (C^2(0, k)[C(-\omega, k - q) + C(\omega, k - q)] - C(-\omega, k - q)C(\omega, k - q)[C(0, k) + C(0, k - 2q)]). \quad (95)$$

Here $F(x) = x^2 / \sinh^2 x$ is the thermal function, which differs from $f(x)$ defined by Eq.(67) by unity. [Recall that the unity in Eq.(67) eliminated the classical component in Eq.(65), which we treated to all orders of perturbation theory.]

With the spectral function of the Coulomb interaction $J(\omega, q)$ given by Eq.(26), the integrals in Eq.(96) can be done analytically in the limit $\gamma \ll T$, since the integral over ω converges at $\omega \sim \gamma$ and one can use the approximation $F(x) \approx 1$. The result is [5]:

$$\Delta\sigma_{deph} = \frac{e^4 DT}{4\pi\gamma^2\sigma_1}. \quad (96)$$

Assuming that the electron–electron interaction is the only mechanism of decoherence, let us determine γ self-consistently. Minimizing the sum $\Delta\sigma_{wl}^{(0)} + \Delta\sigma_{deph}$ with respect to γ , we get γ in agreement with Eq.(56).

The third term in Eq.(92) corresponds to the high frequency contribution to the dephasing term Eq.(94) beyond the approximation $F(x) = 1$ and has the form:

$$\Delta\sigma'_{deph} = -\zeta \left(\frac{1}{2}\right) \frac{e^4 DT}{\pi\gamma^2\sigma_1} \sqrt{\frac{2\gamma}{\pi T}}. \quad (97)$$

The last term is the interaction correction to weak localization and it was presented in Eq.(1).

Let us substitute γ from Eq.(56) into Eq.(92). The first two terms now become of the same order of magnitude and they correspond to the result of [3]. The third term becomes

$$\Delta\sigma'_{deph} = 4\zeta \left(\frac{1}{2}\right) \Delta\sigma_{wl}^{(0)} \sqrt{\frac{2\gamma}{\pi T}} \sim \sqrt{\frac{1}{g(L_{ee})}} \Delta\sigma_{wl}^{(0)}, \quad (98)$$

in agreement with our Eqs.(88,91).

The interaction correction of ref. [5], which we have not considered

$$\Delta\sigma_{cwl} = -3\zeta(3) \frac{e^2\sqrt{D\tau_H}}{2\pi\hbar} \frac{e^2}{2\pi\hbar\sigma_1} \sqrt{\frac{\hbar D}{2\pi T}} \sim \sqrt{\frac{1}{g^3(L_{ee})}} \Delta\sigma_{wl}^{(0)}. \quad (99)$$

The last equality comes from

$$\frac{e^2}{\hbar\sigma_1} \sqrt{\frac{D}{T}} = \frac{e^2\sqrt{D\gamma^{-1}}}{\hbar\sigma_1} \sqrt{\frac{\gamma}{T}} \sim \sqrt{\frac{1}{g^3(L_{ee})}}. \quad (100)$$

Note that this correction is even smaller than $\Delta\sigma_{deph}^{(1)}$.

The reason for the smallness of these corrections is worth emphasizing: the contribution of electromagnetic modes with frequencies greater than the temperature is exponentially suppressed because detailed balance requires that dephasing be produced by the available electronic excitations which have energies smaller than the temperature of the system.

At sufficiently low temperature the semiclassical result of [3] breaks down, because the phase breaking length L_φ becomes large and the dimensionless conductance on that length scale $g(L_\varphi) \sim 1$. In this limit the weak localization picture for a disordered electron system with Coulomb interactions is not applicable and the notion of the dephasing rate becomes irrelevant to the problem of the quantum correction to the conductivity, which is no longer determined by the interference of two time reversed paths.

In summary, we have in this paper dealt with interactions as a mechanism for electronic dephasing by explicitly separating the low and high frequency components. The low frequency, or classical part, was shown to reproduce the previously obtained results [3]. Then we treated the quantum component perturbatively and showed that it contributes negligibly to the conductivity in the regime of weak localization. Finally, we showed that a recent cutoff dependent calculation of quantum corrections [5], originally intended to apply only in the presence of extrinsic phase-breaking effects, is in agreement with ours if the cutoff is interpreted self consistently.

We conclude, that the electron–electron interaction via Coulomb force cannot explain the saturation of the dephasing rate in the weak localization regime. We believe that another mechanism is needed to explain the results of the experimental paper [4].

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APPENDIX A:

We discuss the derivation of the Cooperon in the presence of classical electric field $V(t, \mathbf{r})$. It is convenient to work in space-time representation. The diagram equation for the cooperon is shown in Fig.2. The corresponding analytical equation has the form:

$$C(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}') = C^{(0)}(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}') + \int \dots \int dt_3^+ dt_4^+ dt_3^- dt_4^- d\mathbf{r}_3 d\mathbf{r}_4 C^{(0)}(t_1^+, t_1^-, t_3^+, t_3^-, \mathbf{r}, \mathbf{r}_3) \Sigma^+(t_3^+, t_3^-, t_4^+, t_4^-, \mathbf{r}_3, \mathbf{r}_4) C(t_4^+, t_4^-, t_2^+, t_2^-, \mathbf{r}_4, \mathbf{r}') + \int \dots \int dt_3^+ dt_4^+ dt_3^- dt_4^- d\mathbf{r}_3 d\mathbf{r}_4 C^{(0)}(t_1^+, t_1^-, t_3^+, t_3^-, \mathbf{r}, \mathbf{r}_3) \Sigma^-(t_3^+, t_3^-, t_4^+, t_4^-, \mathbf{r}_3, \mathbf{r}_4) C(t_4^+, t_4^-, t_2^+, t_2^-, \mathbf{r}_4, \mathbf{r}'). \quad (\text{A1})$$

We introduced notations:

$$\begin{aligned} \Sigma^+(t_3^+, t_4^+, t_3^-, t_4^-, \mathbf{r}_3, \mathbf{r}_4) &= \int dt_5 \int d\mathbf{r}_5 G^{(R)}(t_3^+ - t_5, \mathbf{r}_3 - \mathbf{r}_5) V(t_5, \mathbf{r}_5) \\ &\quad G^{(R)}(t_5 - t_4^+, \mathbf{r}_5 - \mathbf{r}_4) G^{(A)}(t_3^- - t_4^-, \mathbf{r}_3 - \mathbf{r}_4) \\ \Sigma^-(t_3^+, t_4^+, t_3^-, t_4^-, \mathbf{r}_3, \mathbf{r}_4) &= \int dt_5 \int d\mathbf{r}_5 G^{(R)}(t_3^+ - t_4^+, \mathbf{r}_3 - \mathbf{r}_4) G^{(A)}(t_3^- - t_5, \mathbf{r}_3 - \mathbf{r}_5) \\ &\quad V(t_5, \mathbf{r}_5) G^{(A)}(t_5^- - t_4^-, \mathbf{r}_5 - \mathbf{r}_4), \end{aligned} \quad (\text{A2})$$

$C^{(0)}(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}')$ is the Cooperon without interaction. In the Fourier representation it is given by a ladder shown in Fig.2:

$$C^{(0)}(\omega, \mathbf{k}) = \frac{1}{2\pi\nu\tau_{imp}} \frac{1}{1 - (2\pi\nu\tau_{imp})^{-1}\Pi_0(\omega, \mathbf{k})} = \frac{1}{2\pi\nu\tau_{imp}} \frac{1}{D\mathbf{k}^2 - i\omega}, \quad (\text{A3})$$

where

$$\Pi_0(\omega, \mathbf{k}) = \int \frac{d\mathbf{p}}{(2\pi)^d} G^{(R)}(\epsilon + \omega, \mathbf{p} + \mathbf{k}) G^{(A)}(\epsilon, \mathbf{p}) = 2\pi\nu\tau_{imp}(1 + i\omega\tau_{imp} - D\mathbf{k}^2\tau_{imp}). \quad (\text{A4})$$

Using the definition of the Fourier transform of the Cooperon, see Eq.(38), it is easy to prove the following relation:

$$C^{(0)}(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}') = \delta(t_1^+ + t_1^- - t_2^+ - t_2^-) \int \frac{d\mathbf{k}}{(2\pi)^d} \int \frac{d\omega}{2\pi} e^{-\omega(t_1^+ - t_2^+ - t_1^- + t_2^-)} e^{i\mathbf{q}(\mathbf{r} - \mathbf{r}')} C^{(0)}(\omega, \mathbf{k}). \quad (\text{A5})$$

Now consider the self energy of the Cooperon, which is the product of Green's functions given by Eq.(A2). In Eq.(A7) we use equations

$$\begin{aligned} \int \frac{d\mathbf{p}}{(2\pi)^d} G^{(R)}(\epsilon, p) G^{(R)}(\epsilon + \omega, \mathbf{p} + \mathbf{q}) G^{(A)}(\epsilon', -\mathbf{p} + \mathbf{k}) &\approx 2\pi i\nu\tau_{imp}^2 \\ \int \frac{d\mathbf{p}}{(2\pi)^d} G^{(A)}(\epsilon, p) G^{(A)}(\epsilon + \omega, \mathbf{p} + q) G^{(R)}(\epsilon', \mathbf{k} - \mathbf{p}) &\approx -2\pi i\nu\tau_{imp}^2, \end{aligned} \quad (\text{A6})$$

to get

$$\begin{aligned}\Sigma^+(t_3^+, t_3^-, t_4^+, t_4^-, \mathbf{r}_3, \mathbf{r}_4) &= 2\pi\nu i\tau_{imp}^2 V(t_4^+, \mathbf{r}_4)\delta(t_3^+ - t_4^+)\delta(t_3^- - t_4^-)\delta(\mathbf{r}_3 - \mathbf{r}_4) \\ \Sigma^-(t_3^+, t_3^-, t_4^+, t_4^-, \mathbf{r}_3, \mathbf{r}_4) &= -2\pi\nu i\tau_{imp}^2 V(t_4^+, \mathbf{r}_4)\delta(t_3^+ - t_4^+)\delta(t_3^- - t_4^-)\delta(\mathbf{r}_3 - \mathbf{r}_4).\end{aligned}\quad (\text{A7})$$

Note, that Eq.(A6) are obtained in the lowest order in $\omega\tau_{imp}$ and $D\mathbf{q}^2\tau_{imp}$.

We see that the property of the Cooperon without interaction given by Eq.(A5) allows us to find the interacting Cooperon in the form

$$C(t_1^+, t_1^-, t_2^+, t_2^-, \mathbf{r}, \mathbf{r}') = \mathcal{C}(t_1^+ + t_1^-, t_1^+ - t_1^-, t_2^+ - t_2^-, \mathbf{r}, \mathbf{r}')\delta(t_1^+ + t_1^- - t_2^+ - t_2^-). \quad (\text{A8})$$

Using Eqs.(A7) and (A8) we obtain the integral equation for the Cooperon:

$$\begin{aligned}\mathcal{C}(T, \eta_1, \eta_2, \mathbf{r}, \mathbf{r}') &= \mathcal{C}^{(0)}(T, \eta_1, \eta_2, \mathbf{r}, \mathbf{r}') + \\ 2\pi\nu i\tau_{imp}^2 \int \int d\eta_3 d\mathbf{r}_3 &\mathcal{C}^{(0)}(T, \eta_1, \eta_3^+, \mathbf{r}, \mathbf{r}_3)(V(T + \eta_3/2, \mathbf{r}_3) - V(T - \eta_3/2, \mathbf{r}_3))\mathcal{C}(T, \eta_3, \eta_2, \mathbf{r}_3, \mathbf{r}').\end{aligned}\quad (\text{A9})$$

This equation can be considered as a Dyson equation for the Cooperon in the classical field.

Since the noninteracting cooperon satisfies

$$\left(\frac{\partial}{\partial\eta} - D\nabla^2\right)\mathcal{C}^{(0)}(\eta, \mathbf{r}) = \frac{1}{2\pi\nu\tau_{imp}^2}\delta(\eta)\delta(\mathbf{r}), \quad (\text{A10})$$

we get the Schrödinger type equation for the interacting Cooperon:

$$\left(\frac{\partial}{\partial\eta} - D\nabla^2 - i\tilde{V}(T, \eta, \mathbf{r}, \mathbf{r}')\right)\mathcal{C}(T, \eta, \eta', \mathbf{r}) = \frac{\delta(\eta - \eta')\delta(\mathbf{r} - \mathbf{r}')}{2\pi\nu\tau_{imp}^2}, \quad (\text{A11})$$

where $\tilde{V}(T, \eta, \mathbf{r}) = V(T + \eta/2, \mathbf{r}) - V(T - \eta/2, \mathbf{r})$. Note that a constant electric field does not influence the Cooperon.

The solution to Eq.(A11) can be represented in path integral form, given by Eq.(42).

APPENDIX B:

Now we will demonstrate how to get Eq.(65) from Eq.(63) for the impurity average value of the conductivity correction.

All diagrams which contribute to the dephasing term (see [5]) can be classified according to Fig.3 into four groups.

First we consider diagrams, which contain the Keldysh component of the electric field Green's function. Since it is coupled to electrons by two diagonal in the Keldysh space vertices, the allowed positions for the Keldysh component of the electron Green's function are situated near the current operator, as it is shown in Fig.3 by circles. Those diagrams are similar to the diagrams which contribute to the leading part of the weak localization correction to conductivity, see Appendix A.

All four diagrams have the similat elements: the Hikami box, difined by Eq.(36), three cooperons which are the solution of Eq.(A1), and two Hikami boxes in the form of triangles, see Eq.(A6)

Note that the vertices do not depend on frequency, and substituting the time representation of the Cooperon, determined by Eq.(38), we get

$$\begin{aligned}\Delta\sigma_K &= \frac{8e^2D}{\pi} \int d\mathbf{r}_1 \int d\mathbf{r}_2 \int d\eta \int_{-\eta}^{+\eta} d\zeta_1 \int_{-\eta}^{+\zeta_1} \mathcal{L}^{(K)}(\omega, \mathbf{r}_1 - \mathbf{r}_2) \times \\ &\mathcal{C}(T, \eta, \zeta_1, \mathbf{r}, \mathbf{r}_1)\mathcal{C}(T, \zeta_1, \zeta_2, \mathbf{r}_1, \mathbf{r}_2)\mathcal{C}(T, \zeta_2, -\eta, \mathbf{r}_2, \mathbf{r}) \times \\ &(\cos\omega(\zeta_2 - \zeta_1) - \cos\omega(\zeta_1 + \zeta_2)).\end{aligned}\quad (\text{B1})$$

This expression contains the contribution in the lowest order in $\epsilon_F\tau_{imp}$ to the dephasing correction to conductivity which has the Keldysh component $\mathcal{L}^{(K)}(\omega, \mathbf{r})$ of the electric field Green's function.

Now we can consider the diagrams, which have the retarded or advanced components of the electric field Keldysh Green's function. The diagrams are represented in Fig.3. In this case there are vertices which are not diagonal in the Keldysh space of electron Green's function.

We take as an example the diagram, of the first type. The analytical expression has the following form:

$$G^{(R)}(\epsilon + \omega'/2, \epsilon_1^+, \mathbf{r}, \mathbf{r}_1)G^{(K)}(\epsilon_1^+ - \omega, \epsilon_2^+, \mathbf{r}_1, \mathbf{r}_2)G^{(R)}(\epsilon_2^+, \epsilon_3 + \omega_{ext}/2, \mathbf{r}_2, \mathbf{r}') \\ \mathbf{j}(\mathbf{r}')\mathbf{A}(\mathbf{r}')G^{(A)}(\epsilon_3 - \omega_{ext}/2, \epsilon - \omega'/2, \mathbf{r}', \mathbf{r})\frac{\hbar(\epsilon_3 + \omega_{ext}/2) - \hbar(\epsilon_3 - \omega_{ext}/2)}{\omega_{ext}}\mathcal{L}^{(R)}(\omega, \mathbf{r}_1 - \mathbf{r}_2). \quad (\text{B2})$$

The Keldysh component of electron Green's function is given by Eq.(24). The dephasing correction to conductivity is given by diagrams with $G_0^{(K)}$ situated at the positions shown in Fig.(3). Their sum is

$$\int d\mathbf{r}_1 \int d\mathbf{r}_2 \int \frac{d\omega}{2\pi} \left(\mathcal{L}^{(R)}(\omega, \mathbf{r}_1 - \mathbf{r}_2) - \mathcal{L}^{(A)}(\omega, \mathbf{r}_1 - \mathbf{r}_2) \right) G^{(R)}(\epsilon + \omega'/2, \epsilon_1^+, \mathbf{r}, \mathbf{r}_1) \\ G^{(R)}(\epsilon_1^+ - \omega, \epsilon_2^+, \mathbf{r}_1, \mathbf{r}_2)G^{(R)}(\epsilon_2^+, \epsilon_3 + \omega_{ext}/2, \mathbf{r}_2, \mathbf{r}')G^{(A)}(\epsilon_3 - \omega_{ext}/2, \epsilon - \omega'/2, \mathbf{r}', \mathbf{r}) \\ \mathbf{j}(\mathbf{r}')\mathbf{A}(\mathbf{r}')h(\epsilon_2 - \omega)\frac{\hbar(\epsilon_3 + \omega_{ext}/2) - \hbar(\epsilon_3 - \omega_{ext}/2)}{\omega_{ext}}. \quad (\text{B3})$$

Here we have added the term with $\mathcal{L}^{(A)}(\omega, \mathbf{r}_1 - \mathbf{r}_2)$ to the dephasing correction to conductivity. The same term has to be subtracted from the interaction correction. This procedure was done in [5], where the interaction correction was calculated for the first time. The corresponding analysis has to be done in the spirit of the present calculations taking the classical field in all orders of perturbation theory.

Now the diagram can be represented as a combination of Cooperons, defined by Eq.(38), Hikami boxes, see Eq.(36) and Eq.(A6). Unlike the terms, which have the Keldysh propagator of the electric field, one vertex depends on electron energy, scattered by the electric field. In this case we cannot perform integration over that energy, and we get a non-instantaneous vertex in terms of the Cooperon coupling to the electric field. We get:

$$\Delta\sigma_I = \frac{4e^2D}{\pi} \int d\mathbf{r}_1 \int d\mathbf{r}_2 \int d\eta \int_{-\eta}^{+\eta} d\zeta_1 \int_{-\eta}^{+\zeta_1} \int_{-\infty}^{+\infty} d\tau (\mathcal{L}^{(R)}(\omega, \mathbf{r}_1 - \mathbf{r}_2) - \mathcal{L}^{(A)}(\omega, \mathbf{r}_1 - \mathbf{r}_2)) \times \\ \mathcal{C}(T, \eta, \zeta_1, \mathbf{r}, \mathbf{r}_1)\mathcal{C}(T, \zeta_1, \zeta_2, \mathbf{r}_1, \mathbf{r}_2)\mathcal{C}(T + \tau/2, \zeta_2 + \tau, \tau - \eta, \mathbf{r}_2, \mathbf{r}) \times \\ \exp(i\omega(\zeta_2 - \zeta_1)) \int \frac{d\epsilon_2}{2\pi} \int \frac{d\epsilon_3}{2\pi} e^{i(\epsilon_2^+ - \epsilon_3)\tau} h(\epsilon_2^+ - \omega) \frac{\hbar(\epsilon_3 + \omega_{ext}/2) - \hbar(\epsilon_3 - \omega_{ext}/2)}{\omega_{ext}}. \quad (\text{B4})$$

The conductivity of the system is determined by the average value with respect to the fluctuations of the electric field, see Eq.(22), we consider the average value of the expression in the second line of Eq.(B4). Substituting the Cooperons in the form of Eq.(42), we get for the interaction term of the cooperon action, see the second term in Eq.(44):

$$S_{int}[\tau, \mathbf{r}(t)] = \frac{2e^2T}{\sigma_1} \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{1}{q^2} e^{i\mathbf{q}(\mathbf{r}(t_1) - \mathbf{r}(t_2))} \\ \left(\int_{-\eta}^{\zeta_2} dt_1 \int_{-\eta}^{\zeta_2} dt_2 (\delta(t_1 - t_2) - \delta(t_1 + t_2)) + \right. \\ \left. \int_{\zeta_2}^{+\eta} dt_1 \int_{\zeta_2}^{+\eta} dt_2 (\delta(t_1 - t_2) - \delta(t_1 + t_2)) + \right. \\ \left. \int_{-\eta}^{\zeta_2} dt_1 \int_{\zeta_2}^{\eta} dt_2 (\delta(t_1 + \tau - t_2) + \delta(t_1 - \tau - t_2) - \delta(t_1 + t_2 + \tau) - \delta(t_1 + t_2 - \tau)) \right) \quad (\text{B5})$$

We replace $S_{int}[\tau, \mathbf{r}(t)]$ by $S_{int}[\tau = 0, \mathbf{r}(t)]$ and consider the difference $\Delta_{int}^I = S_{int}[\tau, \mathbf{r}(t)] - S_{int}[\tau = 0, \mathbf{r}(t)]$ as perturbation. The first order term in Δ_{int}^I is of the same order as the higher order terms in the quantum part of the influence functional and can be neglected for our purposes.

So we can neglect the dependence on τ of the second line in Eq.(B4), and perform integration over τ . After that integrals over energies ϵ_2 and ϵ_3 can be done.

The remaining three diagrams can be similarly calculated. Collecting all terms together, see also Eq.(B1) we get Eq.(65).

APPENDIX C:

In this appendix we discuss a recent paper by [8] Golubev and Zaikin, which obtains a saturating dephasing time in the limit of zero temperature, a results very different from [3] and [5]. In our opinion the difference is due to uncontrolled approximations. It is important to note that the contradiction between [8] and [3] appears for $g(L_\varphi) \gg 1$, in which region the corrections to [3] have been shown in the present paper to be negligible.

Before discussing details, some general remarks are in order. The result of [8] is infrared divergent, see Eq.(76) in [8]. In the present paper, we have shown that a consistent calculation does not require an extrinsic cut-off. This suggests that by dropping terms the authors of [8] have lost some essential physics. We would also like to mention that the authors of [8], have referred to the Caldeira–Leggett model of a quantum particle coupled to environment [7] as a similar model, which has to be treated in the same nonperturbative manner as paper [8] claims to treat the weak localization. However, in the solution of the Caldeira–Leggett model, physical quantities are free of unphysical divergences. For example, Eq.(83) in [15] for the average value of the particle momentum is expressed as an integral with an intrinsic infrared cutoff.

In what follows we shall take some formulas from [8] on faith, and question subsequent approximations. The following expression is used to calculate the conductivity of the electron system:

$$\sigma = -\frac{2e^2}{3m} \int_{-\infty}^0 dt' \int \frac{d\mathbf{p}}{(2\pi)^d} \hat{W}(t') \mathbf{p} \frac{\partial n(\mathbf{p})}{\partial p}, \quad (\text{C1})$$

with $\partial n(\mathbf{p})/\partial \mathbf{p} \approx -\mathbf{v}_F \delta(\epsilon)$. The evolution operator $\hat{W}(t')$ is

$$\hat{W}(t') = U_l(t') \cdot U_r(t'), \quad (\text{C2})$$

where

$$\begin{aligned} U_l(t) &= \text{T exp} \left(-i \int_0^t d\tau H_l(\tau, \mathbf{r}_1, \mathbf{r}_2) \right), \\ U_r(t) &= \text{T exp} \left(i \int_0^t d\tau H_r(\tau, \mathbf{r}_1, \mathbf{r}_2) \right) \end{aligned} \quad (\text{C3})$$

and the Hamiltonians in the interaction picture are:

$$\begin{aligned} H_l(t, \mathbf{r}_1, \mathbf{r}_2) &= -eV^+(t, \mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{1}{2} (\delta(\mathbf{r}_1 - \mathbf{r}_2) - 2\rho(\mathbf{r}_1, \mathbf{r}_2)) eV^-(t, \mathbf{r}_2), \\ H_r(t, \mathbf{r}_1, \mathbf{r}_2) &= -eV^+(t, \mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) + \frac{1}{2} (\delta(\mathbf{r}_1 - \mathbf{r}_2) - 2\rho(\mathbf{r}_1, \mathbf{r}_2)) eV^-(t, \mathbf{r}_1). \end{aligned} \quad (\text{C4})$$

Here $V^+(t, \mathbf{r})$ and $V^-(t, \mathbf{r})$ are the components of the fluctuating electric field.

The conductivity defined by Eq.(C1) has to be averaged over fluctuations of the electric field to reproduce the conductivity of the interacting electron system. As the result of this averaging the authors of [8] obtain the following effective action for the electrons:

$$S[t, \mathbf{r}(t)] = S_R[t, \mathbf{r}] + iS_I[t, \mathbf{r}], \quad (\text{C5})$$

where

$$\begin{aligned} S_I[t, \mathbf{r}, \dot{\mathbf{r}}] &= \frac{e^2}{2} \int_{-t}^{+t} dt_1 \int_{-t}^{+t} dt_2 (2I(t_1 - t_2, \mathbf{r}(t_1) - \mathbf{r}(t_2)) \\ &\quad - I(t_1 + t_2, \mathbf{r}(t_1) - \mathbf{r}(t_2)) - I(-t_1 - t_2, \mathbf{r}(t_1) - \mathbf{r}(t_2))). \end{aligned} \quad (\text{C6})$$

$$\begin{aligned} S_R[\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2] &= \frac{e^2}{2} \int_{-t}^{+t} dt_1 \int_{-t}^{+t} dt_2 (R(t_1 - t_2, \mathbf{r}_1(t_1) - \mathbf{r}_1(t_2))(1 - 2n(\mathbf{p}_1(t_2), \mathbf{r}_1(t_2))) - \\ &\quad R(t_1 - t_2, \mathbf{r}_2(t_1) - \mathbf{r}_2(t_2))(1 - 2n(\mathbf{p}_2(t_2), \mathbf{r}_2(t_2))) + \\ &\quad R(t_1 - t_2, \mathbf{r}_1(t_1) - \mathbf{r}_2(t_2))(1 - 2n(\mathbf{p}_2(t_2), \mathbf{r}_2(t_2))) - \\ &\quad R(t_1 - t_2, \mathbf{r}_2(t_1) - \mathbf{r}_1(t_2))(1 - 2n(\mathbf{p}_1(t_2), \mathbf{r}_1(t_2))). \end{aligned} \quad (\text{C7})$$

We have shifted the limits of time integration, so that the sum of the upper and lower limits is equal to zero. $I(t, \mathbf{r})$ and $R(t, \mathbf{r})$ are given by

$$I(t, \mathbf{r}) = \int \frac{d\omega d^d \mathbf{k}}{(2\pi)^{d+1}} \text{Im} \left(\frac{-4\pi}{k^2 \varepsilon(\omega, \mathbf{k})} \right) \coth \frac{\omega}{2T} e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \quad (\text{C8})$$

$$\begin{aligned}
R(t, \mathbf{r}) &= R_e(t, \mathbf{r}) + R_o(t, \mathbf{r}), \\
R_e(t, \mathbf{r}) &= \int \int \frac{d\omega d\mathbf{k}}{(2\pi)^{d+1}} \operatorname{Re} \frac{4\pi}{k^2 \varepsilon(\omega, \mathbf{k})} \cos \omega t e^{i\mathbf{k}\mathbf{r}} \\
R_o(t, \mathbf{r}) &= \int \int \frac{d\omega d\mathbf{k}}{(2\pi)^{d+1}} \operatorname{Im} \frac{4\pi}{k^2 \varepsilon(\omega, \mathbf{k})} \sin \omega t e^{i\mathbf{k}\mathbf{r}}.
\end{aligned} \tag{C9}$$

Let us consider the conclusions that were drawn from these formulas. The authors of [8] omitted the second and third terms on the right hand side of the expression for the imaginary part of the action $S_I[t, \mathbf{r}(t)]$, Eq.(C6). Their argument for doing this is that the first term increases linearly in time t , while both terms in the second line have slower long time dependence. We agree that the first term is indeed important in the limit of very large t . But the contribution to conductivity is determined by time $t = \tau_\varphi$ which satisfies $S_I(\tau_\varphi) \sim 1$, calling the argument into question.

Indeed, one can see the correspondence between the action Eq.(C6) and the semiclassical calculations of [3], see also section III. The form of Eq.(C8) is similar to that found by [3]. If we use $\operatorname{Im}\epsilon^{-1}(\omega, \mathbf{k}) = \omega/4\pi\sigma$, (see Eq.(74) in [8]), we get

$$S_I[t, \mathbf{r}(t)] \sim \int_{-t}^{+t} dt_1 \int \frac{d\mathbf{k}}{(2\pi)^d} \frac{1}{\mathbf{k}^2} \left(1 - e^{i\mathbf{k}(\mathbf{r}(t_1) - \mathbf{r}(-t_1))} \right) \tag{C10}$$

Here we have used the fact, that the integral over frequencies gives a narrow function of $t_1 - t_2$ which vanishes for $|t_1 - t_2| \geq 1/T$. Then keeping in mind the inequality $T\tau_\varphi \gg 1$ we can approximate that function by $\delta(t_1 - t_2)$.

The unity in the brackets corresponds to the term considered in [8]. [See their Eq.(71).] It originates from the first term in Eq.(C6): The second term in Eq.(C10), which originates from the second line in Eq.(C6) was neglected in [8]. Indeed, one is allowed to neglect it when the exponent $\mathbf{k}(\mathbf{r}(t_1) - \mathbf{r}(t_2))$ is large. However, for the long wavelength ($|\mathbf{k}| \rightarrow 0$) modes which enter the integral in Eq.(C10), one has to keep this term.

The real part of the effective action contains the factor $(1 - 2n(\mathbf{p}, \mathbf{r}))$. The authors of [8] neglected the time dependence of the momentum. We expect that the \mathbf{p} -dependence of the action is crucial for obtaining the correct result. It is worth mentioning that $n(\mathbf{p}, \mathbf{r})$ is a sharp function of momentum near the Fermi surface of the electrons. The approximation of a sharp function by its value at a particular point is at best dangerous. We will present arguments which raise questions about its validity. For this purpose we rewrite the real part of the action S_R , Eq.(C7) in another form:

$$\begin{aligned}
S_R[\mathbf{r}(t), \mathbf{p}(t)] &= \frac{e^2}{2} \int_{-t}^{+t} dt_1 \int_{-t}^{+t} dt_2 (1 - 2n(\mathbf{p}(t_2), \mathbf{r}(t_2))) \\
&\quad (R_o(t_1 - t_2, \mathbf{r}(t_1) - \mathbf{r}(t_2)) - R_o(t_1 + t_2, \mathbf{r}(t_1) - \mathbf{r}(t_2))).
\end{aligned} \tag{C11}$$

Eq.(C11) is derived for the time reversed paths $\mathbf{r}_1(t) = \mathbf{r}_2(-t) = \mathbf{r}(t)$, that is for the same class of trajectories as Eq.(68) in [8]. We represented it in the most transparent form, so that a complicated factor $(1 - 2n(\mathbf{p}(t_2), \mathbf{r}(t_2)))$ is taken at the same moment of time for all terms of the integrand in Eq.(C11). We see, that if $n(\mathbf{p}(t))$ is a constant or an even function of time t , the real part of the effective action vanishes. In the opposite case, when $n(\mathbf{p}(t))$ is odd, the real part of the effective action is of the same order, as the imaginary part, taken at $T = 0$.

Now let us understand if variations of $n(\mathbf{p}(t))$ may be important.

We can introduce the eigenstates of the system without interaction. In the interaction picture we have states $|\epsilon, j, t\rangle$, where ϵ is the energy of the state, j labels the degenerate levels corresponding to ϵ . We have the following equality:

$$\langle \epsilon, j, t | \epsilon', j', t' \rangle = \delta_{\epsilon, \epsilon'} \delta_{j, j'} e^{-i\epsilon(t-t')}. \tag{C12}$$

In the eigenstate basis the density matrix can be represented as

$$\rho = |\epsilon, j, t\rangle \frac{1}{\exp(\epsilon/T) + 1} \langle \epsilon, j, t|. \tag{C13}$$

Then, according to Eq.(C1, C2) we have to evaluate

$$M_{jj'}(\epsilon, \epsilon') = \sum_{|\epsilon_i| \sim T, j_i} \overline{\langle e_{l, j_l, t} | U_l(t) | \epsilon_i, j_i, 0 \rangle \langle \epsilon_i, j_i, 0 | U_r(t) | \epsilon_r, j_r, t \rangle}, \tag{C14}$$

here the sum over the initial states is extended over the energy range, comparable with the temperature of the system and $\overline{(\dots)}$ means average over the electric field fluctuations.

The evolution operators, Eq.(5.13), are understood as an expansion of the exponential functions. Then the right hand side of Eq.5.17 is a series of matrix elements, which we denote as $M_{jj'}^{(n,m)}(\epsilon, \epsilon')$, where n, m shows the order of the expansion of $U_l(t)$ or $U_r(t)$. As an example let us take the third term of the series for $U_l(t)$ and the first term of $U_r(t)$. We have

$$\begin{aligned}
M_{jj'}^{(2,0)}(\epsilon, \epsilon') &= - \sum_{|\epsilon_i| \sim T, j_i} \int_0^t dt_1 \int_0^{t_1} dt_2 \\
&< \epsilon, j, t | \overline{H_l(t_1) H_l(t_2)} | \epsilon_i, j_i, 0 \rangle \langle \epsilon_i, j_i, 0 | \epsilon', j', t \rangle = \\
&\int \int \frac{d\omega d\mathbf{q}}{(2\pi)^{d+1}} \sum_{\epsilon_2, j_2} \sum_{|\epsilon_i| \sim T, j_i} \int d\mathbf{r}_1 \int d\mathbf{r}_2 \\
&\int_0^t dt_1 \int_0^{t_1} dt_2 e^{i\epsilon_r(t-t_1)} e^{i(\epsilon_2+\omega)(t_1-t_2)} e^{i\epsilon_i t_2} \\
&< \epsilon, j, t_2 | e^{i\mathbf{q}\mathbf{r}_1} | \epsilon_2, j_2, t_2 \rangle \langle \epsilon_2, j_2, t_1 | e^{-i\mathbf{q}\mathbf{r}_2} | \epsilon_i, j_i, t_1 \rangle \delta_{\epsilon_i, \epsilon'} \delta_{j_j, j'} \\
&\left(\frac{1}{2} \left(\coth \frac{\omega}{2T} + \tanh \frac{\epsilon_2}{2T} \right) \text{Im} \frac{4\pi}{q^2 \epsilon(\omega, \mathbf{q})} + i \tanh \frac{\epsilon_2}{2T} \text{Re} \frac{4\pi}{q^2 \epsilon(\omega, \mathbf{q})} \right),
\end{aligned} \tag{C15}$$

where $\chi(\omega, \mathbf{q})$ is the susceptibility of the electron system.

The first term in the last line of Eq.(C15) comes from the $\overline{V^+(t)V^+(0)}$ correlation function and the other contain imaginary part of $\overline{V^-(t)V^+(0)}$. $\overline{V^-(t)V^-(0)}$ is identically zero and $\overline{V^+(t)V^-(0)}$ does not contribute to the above matrix element due to causality.

In ref. [8] only the imaginary part of the dielectric susceptibility of the electron system contributes to the weak localization correction. That is why we consider only the part of the matrix element $M_{jj'}^{(2,0)}(\epsilon, \epsilon')$, which contains $\text{Im}\epsilon^{-1}(\omega, \mathbf{q})$. [It was shown in [5], that terms with $\text{Re}\epsilon^{-1}(\omega, \mathbf{q})$ contribute to the interaction correction to weak localization, and do not contribute to the dephasing correction to conductivity.] Since the characteristic values of the upper limits of time integrals in Eq.(C15) are of the order of dephasing time τ_ϕ , the integral over t_2 vanishes, unless $|\epsilon_2 + \omega - \epsilon_i| \tau_\phi < 1$. We have $\tau_\phi T \gg 1$ and consider the high frequencies $\omega \gg T$. Since $\epsilon_i \sim T$, we see that the contribution of high frequencies is exponentially suppressed. One can repeat this analysis for other terms $M_{jj'}^{(n,m)}(\epsilon, \epsilon')$ and observe the same type of cancellation of high frequency contributions to the conductivity in any order of perturbation theory. A similar discussion can be found in [14].

Note, that one can take sum

$$M_{jj'}^{(2,0)}(\epsilon, \epsilon') + M_{jj'}^{(1,1)}(\epsilon, \epsilon') + M_{jj'}^{(0,2)}(\epsilon, \epsilon').$$

This expression reduces to the quantum correction found in [5], but in this case one has to perform a sum over the spectrum of the disordered electron system. The expression would be divergent at $\mathbf{q} \rightarrow 0$. To cure this divergence one should sum all orders of perturbation theory or introduce an external magnetic field to remove the long wave divergence.

We emphasize again that the paper [8] predicts a saturation in the dephasing rate when the inequality $\tau_\phi T \gg 1$ is still satisfied. Our discussion in this appendix is based exactly on the same inequality. And in this case we see that terms in the second line of Eq.(C6) have to be kept and there is no contribution from electromagnetic modes with excitation energy much greater than temperature. In short, the ‘nonperturbative’ calculations of [8] disregarded terms which we have shown to be important.

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