

Comment on PRB 59, 9195 (1999) and 62, 14061 (2000) by D. S. Golubev and A. D. Zaikin.

Golubev and Zaikin have presented calculations¹ which predict a finite dephasing time at zero temperature due to the electron-electron interaction. These calculations are in error because (i) they do not reproduce the result of independent calculations^{2,3}, and (ii) the conclusion of ref. [1] is physically inconsistent^{2,4}. In their recent publications^{5,6} Golubev and collaborators, (GZS) continue to insist that the Coulomb interaction between electrons in the regime of weak localization (WL) leads to a finite dephasing time $\tau_\varphi(T)$ at zero temperature: $\tau_\varphi(0) = \tau_{GZS}$. This contribution comes from the ultraviolet part of the electron-electron interaction and we will focus our discussion on the ultraviolet cutoff in the theory.

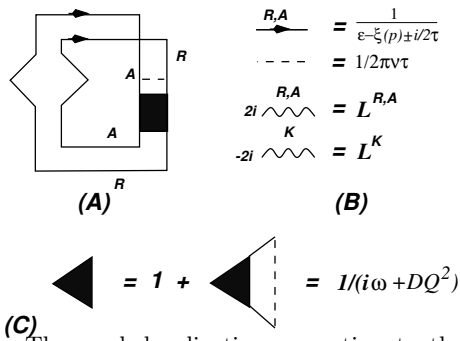


FIG. 1. The weak localization correction to the conductivity is shown in Fig. 1(A). Figure 1(B) represents the basic elements of the diagrammatic technique. The renormalized vertex is the solution to the diagrammatic equation, see Fig. 1(C).

To our opinion, there is now overwhelming experimental evidence against GZS statements. However, we be-

lieve that since the debate is about a well-defined theoretical problem, it has to be resolved by purely mathematical means without invoking the experimental results.

We have already published a detailed discussion of the theoretical side of the problem². Taking into account the broad attention which the discussion attracted, and the current “antiperturbative” sentiments among some condensed matter theorists, we found it is worth writing this comment. We highlight one of numerous mistakes in their calculations to demonstrate that Refs. [1,5,6] presented an incorrect perturbative rather than a non-perturbative treatment of the problem. We address our comment to those experts in the field who still want to get a concrete answer to the question where GZS made the most crucial mistake.

The weak localization correction to the conductivity

$$\delta\sigma_{wl} = -\frac{\sigma_d}{\nu} \int \frac{d\varepsilon d\varepsilon_1 d\varepsilon_2}{(2\pi)^3} \frac{d^d \mathbf{Q}}{(2\pi)^d} \frac{\mathcal{C}(\varepsilon, \varepsilon_1; \mathbf{Q})}{2T \cosh^2 \varepsilon/2T}. \quad (1)$$

is determined by the diagram shown in Fig. 1, where the Cooperon \mathcal{C} is defined in Fig. 2. Here σ_d is the conductivity of a d-dimensional sample and ν is the density of states at the Fermi surface. Terms (a) – (f) in Fig. 2 describe the effect of the electron-electron interaction on the Cooperon. The role of terms (a) in Fig. 2 is to cure the infrared divergence at small energy-momentum transfer ($Q \lesssim 1/L_\varphi$) and thus to restore the gauge invariance^{7,2,3}. These terms do not diverge ultravioletly and thus are not important for our discussion.

For remaining terms (b) – (f) we have

$$\mathcal{C} \begin{pmatrix} \varepsilon_1, \varepsilon_3 \\ \varepsilon_2, \varepsilon_4 \end{pmatrix}; \mathbf{Q} = \frac{(2\pi)^2 \delta(\varepsilon_1 - \varepsilon_3) \delta(\varepsilon_2 - \varepsilon_4)}{-i(\varepsilon_1 - \varepsilon_2) + DQ^2 + \Sigma(\varepsilon_1, \varepsilon_2, Q)}. \quad (2)$$

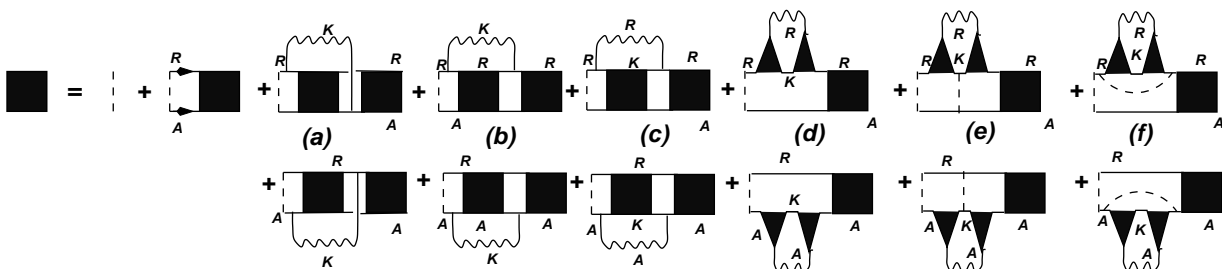


FIG. 2. The diagrammatic equation for the Cooperon \mathcal{C} in the presence of electron-electron interaction. Note that the Cooperon and the vertex on r.h.s. of the equation contain the interaction as well. The basic diagram elements are defined in Figs. 1(B) and (C). The retarded, Keldysh and advanced fermionic propagator are labeled by R,A, and K respectively. In averaging procedure, one uses the relation $G^K(\varepsilon) = [G^R(\varepsilon) - G^A(\varepsilon)] \tanh(\varepsilon/2T)$ for the exact Green functions of disordered system.

The dephasing time τ_φ is determined by $\tau_\varphi^{-1} = \Sigma(\varepsilon, \varepsilon, Q = 0)$, the Cooperon’s self energy at $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $Q = 0$. Expansion in \mathbf{Q} leads to the correction to the diffusion constant, D , free of ultraviolet divergences.

(See Ref. [2] for details).

Taking only term (b) in Fig. 2 into account,

$$\Sigma^{(b)}(\varepsilon, \varepsilon, 0) = \text{Re} \int \frac{d\omega}{\pi} \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{\text{Im} L^A(\omega, \mathbf{q})}{(-i\omega + D\mathbf{q}^2)} \coth \frac{\omega}{2T}, \quad (3)$$

we encounter an ultraviolet divergence and $\Sigma^{(b)}(\varepsilon, \varepsilon, 0)$ reproduces τ_{GZS} from Eq. (71) of Ref. [1] and Eq. (13) of Ref. [6]. We used the following relation between the Keldysh component, $L^K(\omega, \mathbf{q})$, and the retarded (advanced) components $L^{R(A)}(\omega, \mathbf{q})$ of the screened interaction propagator: $L^K(\omega, \mathbf{q}) = \coth(\omega/2T)(L^R(\omega, \mathbf{q}) - L^A(\omega, \mathbf{q}))$.

However, the contribution

$$\Sigma^{(c-f)}(\varepsilon, \varepsilon, 0) = \text{Re} \int \frac{d\omega}{\pi} \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{\text{Im} L^A(\omega, \mathbf{q})}{(-i\omega + D\mathbf{q}^2)} \tanh \frac{\varepsilon - \omega}{2T}. \quad (4)$$

exactly cancels out this ultraviolet divergency. Thus, the self-energy $\Sigma = \Sigma^{(b)} + \Sigma^{(c-f)}$ is determined by $|\omega| \lesssim T$ (according to Eq. (1) $\varepsilon \leq T$).

Comparing Eqs. (3) and (4) with the result of Ref. [1], we conclude the contribution of diagram (b) in Fig. 2 is included in the result of ref. [1,6], (see Eq. (71) there), whereas the contribution of diagrams (c)-(f) in Fig. 2 is omitted in all orders of perturbation theory in Ref. [1].

In the framework of GZS's approach, the contributions from diagrams (a)-(b) can be associated with their S_I , and those from (c)-(f) with their iS_R . The reason why GZ's results don't contain any contributions from (c)-(f) is because they simply neglect the contribution of iS_R to the dephasing rate. They try to justify this (i) by claiming in Ref.[5,6] that S_R is purely real for *all* paths, and hence cannot contribute to dephasing, and (ii) by claiming in Ref.[1,5,6] that $S_R = 0$ along classical time-reversed paths, and hence can be neglected in the exponent of their path integral when the latter is evaluated in the saddle-point approximation (in which only classical paths are considered). Finally, they claimed in Ref.[5,6] that (iii) they can reproduce perturbation theory by expanding the exponent of their path integral.

All three claims are incorrect: (i) GZS obtain a real expression for S_R because in Ref. [1], because Eq.(43) of Ref.[1] neglects Poisson brackets that are needed when Fourier transforming the $\rho_V V^-$ and $V^- \rho_V$ terms in Eq.(40)⁸. If the Poisson brackets are included, S_R contains an imaginary part (which in perturbation theory ensures that the ultraviolet divergency in iS_R cancels that of S_I).

(ii) GZS obtain $S_R = 0$ along classical time-reversed paths, because in Eq.(43) of Ref. 1 they also [in addition to neglecting Poisson brackets], replace⁹ the density matrix by its "Wigner transform",

$$\rho_{1'4} \stackrel{\text{GZS}}{=} \int \frac{d^3 p}{(2\pi)^3} e^{ip(r_{1'} - r_4)} n \left(H \left(p, \frac{r_{1'} + r_4}{2} \right) - \mu \right), \quad (5)$$

This replacement is unjustified, as will be shown below, and leads them erroneously to conclude that $S_R = 0$.

Claim (iii) is false, because after neglecting Poisson brackets, and using the "Wigner transform Eq. (5)", it is impossible to reproduce the perturbation result in any order, as it will be shown below.

Let us now retrace the argument by which GZS claim in Ref. 5 that they can reproduce perturbation theory: GZS notice correctly that the diagrams (c)-(f) can be rewritten in coordinate representation as an amputated average [compare to Ref. [5], Eq. (A1,A2)]

$$2\pi\nu\tau^2 \Sigma_{12}^{(c-f)}(\varepsilon, \varepsilon) = -\text{Im} \int d\mathbf{r}_3 d\mathbf{r}_4 \int \frac{d\omega}{2\pi} L_{34}^R(\omega) \times \langle G_{13}^R(\varepsilon) G_{34}^K(\varepsilon - \omega) G_{42}^R(\varepsilon) G_{12}^A(\varepsilon) \rangle_{\text{amp}}, \quad (6)$$

where the Cooperon legs corresponding to the average in Eq. (6) are cut, coefficient $2\pi\nu\tau^2$ is introduced to match with expressions (3) and (4), and $G_{34}^K(\varepsilon)$ is the Keldysh Green function

$$G_{34}^K(\varepsilon) = \tanh \frac{\varepsilon}{2T} [G_{34}^R(\varepsilon) - G_{34}^A(\varepsilon)]. \quad (7)$$

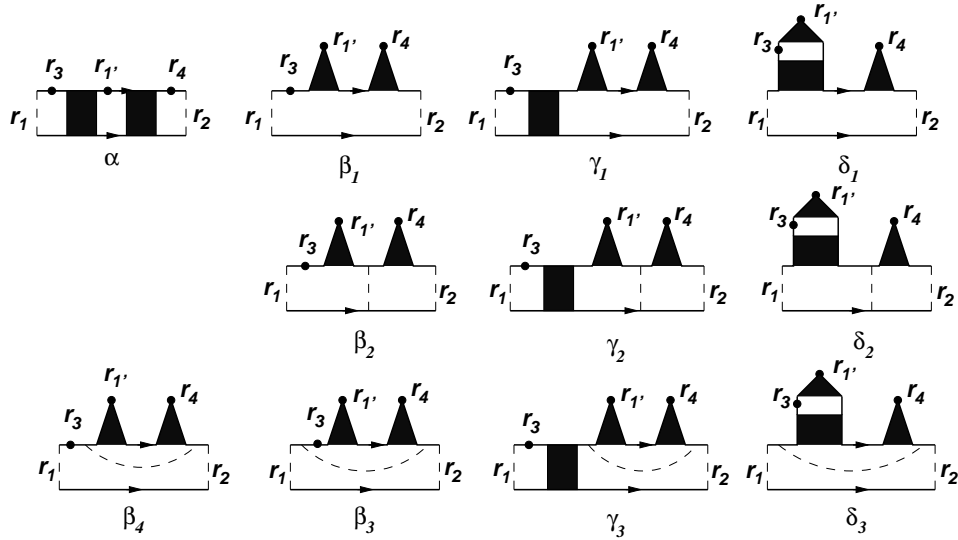


FIG. 3. Diagrams for calculation of $\langle G_{13}^R(\varepsilon)G_{31'}^R(\varepsilon - \omega)[\delta_{1'4} - 2\rho_{1'4}]G_{42}^R(\varepsilon)G_{12}^A(\varepsilon) \rangle_{\text{amp}}$

GZS also present correctly [see Ref. 5, Eq. (A3)]

$$G_{34}^K(\varepsilon) = \int d\mathbf{r}_{1'} (G_{31'}^A(\varepsilon) - G_{31'}^R) [\delta_{1'4} - 2\rho_{1'4}] \quad (8)$$

where $\delta_{1'4} \equiv \delta(\mathbf{r}_{1'} - \mathbf{r}_4)$ and

$$\delta_{1'4} - 2\rho_{1'4} = \int \frac{d\varepsilon}{2\pi i} G_{1'4}^K(\varepsilon) \quad (9)$$

is related to the density matrix $\rho_{1'4}$ of a non-interacting but *disordered* system. Substituting Eq. (8) into Eq. (6) and using the causality principle (analytical properties of the integral over ω) GZS obtain

$$2\pi\nu\tau^2 \Sigma_{12}^{(c-f)}(\varepsilon, \varepsilon) = \text{Im} \int d\mathbf{r}_3 d\mathbf{r}_4 \int d\mathbf{r}_{1'} \frac{d\omega}{2\pi} L_{34}^R(\omega) \quad (10)$$

$$\times \langle G_{13}^R(\varepsilon)G_{31'}^R(\varepsilon - \omega)[\delta_{1'4} - 2\rho_{1'4}]G_{42}^R(\varepsilon)G_{12}^A(\varepsilon) \rangle_{\text{amp}}.$$

Equation (10) still contains all of the relevant contributions, which, if they were calculated correctly (see below) would reproduce Eq. (4). It is important to emphasize, that the density matrix $\rho_{1'4}$ in Eq. (10) is a non-local operator constructed of Fermion operators taken at coincident moments of time but at different space points. For a disordered system, the density matrix ρ is a random long range object; the characteristic scale after which $\rho_{1'4}$ can be considered to be local is $|r_{1'} - r_4| \simeq \sqrt{D/T}$. Since the electron Green's functions $G^{\text{R(A)}}$ are also random and long range, the procedure of impurity averaging is not trivial, see below.

We insist that GZS's uncontrollable approximation Eq.(5) of replacing the density matrix by its "Wigner transform", which they used to argue that S_R does not contribute to the dephasing rate, is impermissible. Once this approximation is used in Eq. (10), it is impossible to recover the correct result in any order of the perturbation theory: the "Wigner transform" neglects all the

contributions to $\rho_{1'4}$ from all electron trajectories except the straight line connecting points 1' and 4. On the other hand, those contributions are included in Eq. (3). As the result of the frivolous replacement (5), GZS calculate two terms entering into one physical quantity (and canceling each other), namely iS_R and S_I in their language, with different accuracy and arrive to their conclusions about zero temperature dephasing! It is neither a new physics, nor a non-perturbative treatment - it is just an incorrect evaluation of a perturbative contribution (10).

The last point we want to clarify is how to obtain Eq. (4) from Eq. (10). A correct way is to use Eqs. (9) and (10) and perform the disorder averaging according to standard rules, see Fig. 3. A straightforward calculation (see Appendix) gives Eq. (4). Thus, the GZS's approach to introduce the density matrix into Eq. (6) instead of the Keldysh Green's function generates eleven diagrams out of original four. As a result of neglecting non-locality of the density matrix and substitution Eq. (5) all those diagrams are lost, i.e. scattering on all the impurities between points 1' and 4 is not taken into account, compare Fig. 3 and Fig. 3 in Ref. [5].

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⁸ We are thankful to J. von Delft for enlightening this particular point in GZS derivation.

⁹ Compare Eqs. (40) by Eqs. (43) in ref. [1], and Eqs. (A18) and (A23) in ref. [5]. See also text above Eq. (A23) in Ref. [5], where the difference between the exact density matrix and its ‘‘Wigner transformation’’ is explicitly disregarded. See also Eq. (29) and discussion after Eq. (31) of Ref. 6, Hikami boxes appearing in Fig. 3 are falsely argued to vanish in Sec. IV of Ref. 6.

APPENDIX

In this appendix we consider diagrams, shown in Fig. 3 and demonstrate that Eq. (10) leads to Eq. (4). We insert Eq. (9) into Eq. (10) to represent Eq. (6) in terms of the exact Green’s functions, defined for a particular disorder realization. We have

$$\begin{aligned} \Sigma_{12}^{(c-f)}(\varepsilon, \varepsilon) &= \text{Im} \int \frac{d\omega d\varepsilon'}{i(2\pi)^2} \frac{d^d \mathbf{q}}{(2\pi)^d} \tanh \frac{\varepsilon'}{2T} L^R(\omega, \mathbf{q}) \\ &\times \Phi_{12}(\mathbf{q}, \omega, \varepsilon, \varepsilon'), \end{aligned} \quad (11)$$

where

$$\Phi_{12}(\mathbf{q}, \omega, \varepsilon, \varepsilon') = \int d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{r}_1' e^{i\mathbf{q}(\mathbf{r}_3 - \mathbf{r}_4)} \times \quad (12)$$

$$\langle G_{13}^R(\varepsilon) G_{31'}^R(\varepsilon - \omega) [G_{1'4}^R(\varepsilon') - G_{1'4}^A(\varepsilon')] G_{42}^R(\varepsilon) G_{12}^A(\varepsilon) \rangle_{\text{amp.}}$$

Following the standard technique for calculation of the disorder average $\Phi(\mathbf{q})$ we obtain diagrams, shown in the Fig. 3.

The disorder averaged Green’s function is

$$\bar{G}^{R,A}(\varepsilon, \mathbf{p} + \mathbf{q}) = \frac{1}{\varepsilon - \xi_{\mathbf{p}} + \mathbf{v}_F \mathbf{q} \pm i/2\tau}, \quad (13)$$

where τ is the mean free path time, $\xi_{\mathbf{p}} = \mathbf{p}^2/2m - \mu$ is the energy, corresponding to momentum \mathbf{p} , μ is the chemical potential and \mathbf{v}_F is the vector of the Fermi velocity directed along the momentum \mathbf{p} .

The diffuson \mathcal{D} and the Cooperon \mathcal{C} for non-interacting system (shown by black boxes) are given by

$$\mathcal{D}(\omega, \mathbf{q}) = \mathcal{C}(\omega, \mathbf{q}) = \frac{1}{2\pi\nu\tau^2} \frac{1}{D\mathbf{q}^2 - i\omega}, \quad (14)$$

and the renormalized vertex Γ (black triangular) is

$$\Gamma(\omega, \mathbf{q}) = \frac{1}{(D\mathbf{q}^2 - i\omega)\tau}. \quad (15)$$

Now we are ready to write down the result for diagrams in Fig. 3. For the $\mathbf{Q} = 0$ component of the Fourier transform

$$\Phi(\mathbf{q}, \omega, \varepsilon, \varepsilon', \mathbf{Q}) = \int \Phi_{12}(\mathbf{q}, \omega, \varepsilon, \varepsilon') e^{i\mathbf{Q}(\mathbf{r}_1 - \mathbf{r}_2)} d(\mathbf{r}_1 - \mathbf{r}_2)$$

we obtain

$$\Phi_{(\mathbf{Q}=0)}^\alpha = 2i\pi\nu\tau^2 \frac{1}{Dq^2 - i(\varepsilon' - \varepsilon)} \frac{1}{Dq^2 + i\omega} \quad (16a)$$

$$\Phi_{(\mathbf{Q}=0)}^\beta = -2\pi\nu i\tau^2 \frac{1}{Dq^2 - i(\varepsilon - \varepsilon')} \frac{1}{0 - i(\varepsilon - \omega - \varepsilon')} \quad (16b)$$

$$\Phi_{(\mathbf{Q}=0)}^\gamma = 2\pi\nu i\tau^2 \frac{1}{0 - i(\varepsilon - \omega - \varepsilon')} \quad (16c)$$

$$\times \left[\frac{1}{Dq^2 - i(\varepsilon - \varepsilon')} + \frac{1}{Dq^2 + i\omega} \right]$$

$$\Phi_{(\mathbf{Q}=0)}^\delta = 2\pi\nu i\tau^2 \frac{1}{Dq^2 - i(\varepsilon - \varepsilon')} \frac{1}{0 - i(\varepsilon - \omega - \varepsilon')} \quad (16d)$$

The sum of diagrams of β and δ -type vanishes and for the sum of α and γ type diagrams we obtain

$$\Phi_{(\mathbf{Q}=0)} = \frac{4\pi\nu i\tau^2}{0 - i(\varepsilon - \omega - \varepsilon')} \text{Re} \left[\frac{1}{Dq^2 - i(\varepsilon - \varepsilon')} \right] \quad (17)$$

$$= 4\pi\nu i\tau^2 \text{Re} \left(\frac{1}{Dq^2 - i(\varepsilon - \varepsilon')} \right) \quad (18)$$

$$\times \left[2\pi\delta(\varepsilon - \varepsilon' - \omega) + \frac{1}{-0 - i(\varepsilon - \varepsilon' - \omega)} \right].$$

The first term in the last line is just a δ -function, which allows us to perform integration over ε' . The second term vanishes after integration over ω due to analytical properties of $L^R(\omega)$ [causality principle]. As a result, we obtain Eq. (4).