Statistical Mechanics, Physics 715
Mid-term exam, due October 26, 2009

Problem 1. Evaluate the load that a helium balloon can carry at elavation 5000 ft, if it can reach 12000 ft without the load. Assume that it has a cylindrical shape with radius 20 ft and hight is 5 ft. You can make reasonable assumptions about the atmosphere or use handbook/online information about it.

Problem 2. In general, the free energy of a three dimensional Boltzmann gas in volume V is given by the following equation

$$F = -NT \ln \left[\frac{eV}{N} \int e^{-\varepsilon(\boldsymbol{p})/T} \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} \right]. \tag{1}$$

Find the temperature dependence of the free energy of a Boltzmann gas with kinetic energy of particles $\varepsilon(\boldsymbol{p}) = c|\boldsymbol{p}|^4$. What are the entropy, internal energy, heat capacity and equation of state of this gas?

Problem 3. Calculate the anharmonic correction to the vibrational heat capacity at temperature $T \gtrsim \hbar\omega_0$ of an ideal gas with a single vibrational degree of freedom described by an one-dimensional Hamiltonian $H = p^2/2m + m\omega_0^2 x^2/2 + \alpha x^4/\lambda_q^4$, where $\lambda_q = \sqrt{\hbar/m\omega_0}$ and $\alpha \ll 1$. What is the limit of applicability of your calculations.