Statistical Mechanics, Physics 715
Homework Assignment 4, due December 7, 2009

Problem 1. Calculate the paramagnetic and diamagnetic susceptibilities of an ideal three dimensional electron gas at high temperatures $T \gg E_{\rm F}$, where $E_{\rm F}$ is the Fermi energy.

Problem 2. Consider a system of N classical magnetic moments $\boldsymbol{\mu}$ with vector $\boldsymbol{\mu}$ constrained to a two-dimensional plane. In the presence of in-plane magnetic field H, the energy of the system is

$$E = -\mu H \sum_{i=1}^{N} \cos\phi_i,$$

where ϕ_i is the angle between $\boldsymbol{H}$ and $\boldsymbol{\mu}$. Find the induced magnetic moment and magnetic susceptibility. At high temperatures the susceptibility has the Curie's law $\chi = \alpha/T$. Evaluate the value of α.

Hint:

$$\int_0^{2\pi} \exp(x\cos\theta)d\theta = 2\pi I_0(x),$$

where $I_0(x)$ is the Bessel function and $I_0(x) \approx 1 + x^2/4$ for $|x| \ll 1$.

Problem 3. Problem 10.5 from Huang, 2nd edition.

Problem 4. A model of dilute magnetic impurities. A system of magnetic spin 1/2 impurities consists of non-interacting impurities with concentration n_1 and impurity pairs with concentration n_2. The interaction between impurities in a pair is described by the Hamiltonian $\hat{H}_{\rm pair} = J\hat{\vec{s}}_1 \cdot \hat{\vec{s}}_2$.
a) Calculate the heat capacity if J for all pairs are equal, consider both cases of $J > 0$ and $J < 0$.
b) Calculate magnetic moment of this system as a function of the applied magnetic field B.
c) Calculate the heat capacity and magnetic susceptibility of this system, if J for all pairs are equal in magnitude, but the sign of J is positive or negative with equal probability.

Problem 5. Problem 17.2 from K. Huang.

Problem 6. Spins taking three values $S_i = \pm 1, 0$ are placed on a square $d-$dimensional lattice. The Hamiltonian is

$$\mathcal{H} = J\sum_{\langle ij\rangle} S_i S_j - h\sum_i S_i,$$

where $\langle ij\rangle$ stands for nearest neighboring pairs.
(a) Write an equation for the average value of the magnetization in the mean field approximation.
(b) Calculate the critical temperature in this approximation?
(c) Describe the behavior of magnetization near the critical temperature?
(d) Calculate heat capacity as a function of h and T.