Statistical Mechanics, Physics 715
Homework Assignment 2, due October 19, 2009

Problem 1. For a system considered in Problem 5 of Homework Assignment 1, the exact expression for the free energy has the form

$$F(T) = -NT \log\left(2\cosh\frac{\Delta}{2T}\right).$$

Calculate the average and variance of polarization m, where m is the difference between the numbers of two level systems in ground and excited states.

Problem 2. The pressure of a gas on a wall can be calculated as the change of molecule momentum per unit of time and per unit of the wall area. Using the Maxwell distribution function for velocities of gas molecules, calculate pressure as a function of temperature.

Problem 3. Consider a system with the Helmholtz free energy in the form

$$F(T,V) = Nf(T) - NT\log\frac{V - Nb}{N} - \frac{N^2 a}{V}.$$

Here $f(T)$ is an arbitrary function of temperature, a and b are constants.

a) Obtain the equation of state for this system.

b) Calculate the entropy of this system S and its internal energy E.

c) Find the heat capacity of the system in isochoric, C_V, and isobaric, C_P, processes.

d) For the case of $f(T) = -CT\log T + DT$, calculate the work performed by the system in the adiabatic expansion from volume V_1 to V_2, if initial system temperature was T_1.

Problem 4. The rotational spectrum of two-atom molecules consists of energy levels

$$E_{l,m} = \frac{\hbar^2 l(l+1)}{2I}; \quad l = 0, 1, \ldots; \quad |m| \le l.$$

a) Calculate the leading term of the rotational heat capacity of an ideal gas of N molecules at low temperature $T \ll \hbar^2/I$.

b) Use the Euler-MacLaurin summation formula

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} f(x)dx + \frac{1}{2}f(0) - \frac{1}{12}f'(0) + \frac{1}{720}f'''(0) + \ldots$$

to calculate the first three terms of a high temperature expansion of the partition function $Z = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\exp(-E_{lm}/T)$.

c) Use the result for Z at high temperatures to find the first two terms in the high temperature expansion of the rotational heat capacity.

Problem 5. Calculate the anharmonic correction to the vibrational heat capacity of an ideal gas with a single vibrational degree of freedom described by an one-dimensional Hamiltonian $H = p^2/2m + m\omega_0^2 x^2/2 + \beta x^3/3$ at high temperatures $T \gg \hbar\omega_0$.

Problem 6. Find the partition function, the free energy and the specific heat of an ideal gas in the extreme relativistic case, when the energy of a particle is linear in its momentum: $\varepsilon = c|\vec{p}|$. Also, obtain the result for the specific heat from the equipartition theorem.