

Statistical Mechanics, Physics 715
Final Exam, due December 15, 2009

Problem 1. *10 points* Consider a gas of N non-interacting atoms in a d -dimensional box of volume L^d with kinetic energy of atoms

$$E = \sum_{i=1}^N \eta |\mathbf{p}|^s,$$

where \mathbf{p} is the momentum of i th particle.

(a) Calculate the classical partition function $Z(N, T)$ at temperature T .

(b) Calculate the pressure and the internal energy of this gas. Formulate the equipartition theorem for the non-quadratic degrees of freedom.

Problem 2. *20 points* Write down the expression for the partition function of the hydrogen atom and show that the expression diverges at large values of the principal quantum number. Consider then the atom placed in a cubic box of size L . In this case you can introduce a cutoff M in the summation over the principal quantum numbers. What is the value of M ? Evaluate the temperature at which the atom is in the ground state with probability $1/2$.

Problem 3. *20 points* Consider the free energy in the form

$$F = -\alpha_1 \frac{\phi_1^2}{2} - \alpha_2 \frac{\phi_2^2}{2} + A\phi_1^4 + 2B\phi_1^2\phi_2^2 + C\phi_2^4,$$

where $\phi_{1,2}$ are real and all coefficients, $\alpha_{1,2}$, A , B and C are positive. Find the extremes of the free energy and identify the one which corresponds to the global minimum. Construct the phase diagram in $\alpha_{1,2}$ plane for this system and describe the phase transitions between the phases.