Lecture 16

Reflection coefficients are identical for right and left movers.

We have two asymptotes:

$$\psi(x) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x} \qquad x \to -\infty$$

$$\psi(x) = \underline{\underline{A_2 e^{ik_2 x}}} + B_2 e^{-ik_2 x} \qquad x \to +\infty.$$

Because $\psi(x)$ is a solution to linear differential equation,

$$A_2 = \alpha A_1 + \beta B_1 \qquad \text{"} B_2 = \gamma A_1 + \delta B_1 \text{"}$$

For a stationary Schrödinger equat.

$$\psi^* = A_2^* e^{-ik_2 x} + \underline{\underline{B_2^* e^{ik_2 x}}}$$ is also a solution at $x \to +\infty$.

$$B_2^* = \alpha B_1^* + \beta A_1^* \quad \text{or} \quad B_2 = \alpha^* B_1 + \beta^* A_1$$

$B_2 = 0$ for right movers $\quad \frac{B_1}{A_1} = -\frac{\beta^*}{\alpha^*} \quad R = \left|\frac{\beta}{\alpha}\right|$

$A_1 = 0$ for left movers $\quad \frac{A_2}{B_2} = \frac{\beta}{\alpha^*}; \quad R = \left|\frac{\beta}{\alpha}\right|.$

16.2/ Theorem:

In one-dimensional system, the bound states are not degenerate,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U(x)) = 0.$$

Let us assume that there are two solutions $\psi_1(x)$ and $\psi_2(x)$.

$$\frac{\psi_1''(x)}{\psi_1(x)} = \frac{2m}{\hbar^2}(E-U(x)) = \frac{\psi_2''(x)}{\psi_2(x)}$$

$\psi_1'\psi_2 - \psi_2'\psi_1 = \text{const}$. This constant is zero, since $\psi_2(\pm\infty) = \psi_1(\pm\infty) = 0$.

$$\frac{\psi_1'}{\psi_1} = \frac{\psi_2'}{\psi_2} \quad \text{OR} \quad \psi_1(x) = \psi_2(x)\cdot C,$$

$$\psi_1(x) \equiv \psi_2(x).$$

For a stationary bound state $j(x) = \text{const} = j(\pm\infty) = 0$, The phase is constant $\psi(x) = \sqrt{\rho(x)}\, e^{i\varphi(x)}$;

$\varphi(x) = \text{const}$ and can be choosen to be equal to <u>zero</u>.

16.2

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad \text{for} \quad x < -a$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi(x) = C \cdot \sin lx + D \cos lx \quad \text{with}$$

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

$$\psi(x) = Fe^{ikx} \quad \text{for } x > a.$$

There are four boundary conditions

$$Ae^{-ika} + Be^{ika} = -C\sin la + D\cos la$$

$$ik[Ae^{-ika} - Be^{ika}] = l[C\cos la + D\sin la]$$

$$C\sin la + D\cos la = Fe^{ika}$$

$$l[C\cos la - D\sin la] = ik\,Fe^{ika}$$

$$B = i\,\frac{\sin 2la}{2lk}\,(l^2 - k^2)F$$

$$F = \frac{e^{-2ika}A}{\cos 2la - i\,\frac{k^2+l^2}{2kl}\sin 2la}$$

$$T = \frac{|F|^2}{|A|^2} = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)}\sin\frac{2a}{\hbar}\sqrt{2m(E+V_0)}}$$

16.4.

If $E < |V_0|$ and $V_0 > 0$

$$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2 \frac{2a}{\hbar} \sqrt{2m(V_0 - E)}}$$

$$T \propto e^{-\frac{4a}{\hbar}\sqrt{2m(V_0 - E)}}$$

for the amplitude $e^{-\frac{2a}{\hbar}\cdot\sqrt{2m(V_0-E)}}$

$$= e^{-\frac{2a \cdot |p|}{\hbar}}$$

with imaginary p!