Many-Body Physics and $\beta\beta$ Decay

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Light Neutrinos

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \begin{pmatrix}
\mathcal{U}_\nu \\
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}$$

\(\mathcal{U}\) contains three mixing angles (and a few phases).

From oscillation experiments:

- **Solar \(\nu\)’s:**
  \[\Delta m^2_{\text{sol}} \approx 8 \times 10^{-5} \text{ eV}^2\]
  \[\theta_{\text{sol}} \approx 34^\circ\]

- **Atmospheric \(\nu\)’s:**
  \[\Delta m^2_{\text{atm}} \approx 2 \times 10^{-3} \text{ eV}^2\]
  \[\theta_{\text{atm}} \approx 45^\circ\]

- **Reactor \(\nu\)’s:**
  Third mixing angle \(\theta_{13}\) is small.
  Atmospheric \(\nu_e\)’s don’t oscillate.
Some of What We Don’t Know

**Majorana or Dirac**
Some of What We Don’t Know

- Majorana or Dirac
- Normal or inverted

Diagram:

- $\theta_{13}$
- CP phases
Some of What We Don’t Know

- Majorana or Dirac
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- Overall mass scale
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- \( \theta_{13} \) and CP phases
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Diagram:

- Mass (eV) scale
- $\nu_3$ and $\nu_1$ with uncertain mass values
- Solar and Atmospheric neutrinos

Questions:

- $\nu_2$ and $\nu_3$ mass comparison
- CP phases
Neutrinoless Double-Beta Decay

If energetics are right (ordinary beta decay forbidden) and neutrinos are Majorana, we can observe two neutrons turning into protons, emitting two electrons and nothing else. This is different from the already observed $2\nu$ process.

$Z, N \rightarrow Z+1, N-1 \rightarrow Z+2, N-2$
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Light-$\nu$-exchange amplitude proportional to “effective mass”

$$m_{ee} \equiv \sum_i m_i U_{ei}^2$$
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How Effective Mass Gets into Rate

\[
[T_{1/2}^{0\nu}]^{-1} = \sum_{\text{spins}} \int |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} - Q_{\beta\beta}) \frac{d^3 p_1}{2\pi^3} \frac{d^3 p_2}{2\pi^3}
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\(Z_{0\nu}\) contains lepton part

\[
\sum_k \bar{e}(x) \gamma_\mu (1 - \gamma_5) U_{e_1} \nu_k(x) \bar{\nu}_k^c(y) \gamma_\nu (1 + \gamma_5) U_{e_1} e^c(y),
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where \(\nu\)'s are Majorana mass eigenstates.
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\[ \sum_k \bar{\epsilon}(x) \gamma_\mu (1 - \gamma_5) U_{ek} \nu_k(x) \overline{\nu}_k^c(y) \gamma_\nu (1 + \gamma_5) U_{ek} e^c(y) , \]

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Contraction gives neutrino propagator:

\[ \sum_k \bar{\epsilon}(x) \gamma_\mu (1 - \gamma_5) \frac{q^\rho \gamma_\rho + m_k}{q^2 - m_k^2} \gamma_\nu (1 + \gamma_5) e^c(y) U_{ek}^2 , \]
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\]

The \(q^\rho \gamma_\rho\) part vanishes in trace, leaving a factor

\[
m_{ee} \equiv \sum_k m_k U_{ek}^2.
\]
Hadronic Part of Amplitude

Integral over times produces a factor

$$\sum_n \frac{\langle f|J^\mu_L(\vec{x})|n\rangle\langle n|J^\nu_L(\vec{y})|i\rangle}{q^0(E_n + q^0 + E_{e2} - E_i)} + (\vec{x}, \mu \leftrightarrow \vec{y}, \nu),$$

with $q^0$ the virtual-\(\nu\) energy and \(J\) the weak current.

$$\langle p|J^\mu(x)|p'\rangle = e^{iqx}u(p) (g V(q^2)\gamma^\mu - g A(q^2)\gamma^5)\gamma^\mu - ig M(q^2)\sigma_{\mu\nu}q^\nu 2m_p + g P(q^2)\gamma^5 q^\mu)u(p').$$

$q^0$ typically $\sim 2 fm^{-1} \approx 100$ MeV, so denominator roughly constant and sum done in closure.
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\[ \langle p|J^\mu(x)|p'\rangle = e^{iq^x}\bar{u}(p) \left( g_V(q^2)\gamma^\mu - g_A(q^2)\gamma_5\gamma^\mu \right. \]

\[ \left. - ig_M(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p} + g_P(q^2)\gamma_5 q^\mu \right) u(p'). \]
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Final Form of Nuclear Part

\[ M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \ldots \]
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with

\[ M_{0\nu}^{GT} = \langle f| \sum_{a,b} H(r_{ab}, \bar{E}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ |i\rangle + \ldots \]

\[ M_{0\nu}^F = \langle f| \sum_{a,b} H(r_{ab}, \bar{E}) \tau_a^+ \tau_b^+ |i\rangle + \ldots \]

\[ H(r, \bar{E}) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \bar{E} - (E_i + E_f)/2} \]
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Corrections (“forbidden” terms, weak form factors) \( \lesssim 30\% \).
Calculating the Matrix Elements

It’s hard, because

- Relevant nuclei heavy ($\tilde{A} > 75$) and complicated. No controlled approximation schemes such nuclei.
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- $M_{fi}$ small and sensitive to delicate two-body space/spin correlations.
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Eventually (really!) will have accurate numerical calculation. In meantime, chip away at problem.
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A few in the shell model.
Long History of Calculations

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**QRPA vs. Shell Model:**

(protons) (neutrons)
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- Large single-particle space; simple correlations.
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**QRPA vs. Shell Model:**

Small single-particle space; arbitrary correlations within it.
Current Situation

![Graph showing data points for NSM (Jastrow) and (R)QRPA (Jastrow, UCOM) with elements Ge, Se, Zr, Mo, Cd, Te, and Xe on the x-axis and the function $\langle M', 0\nu \rangle$ on the y-axis.]
Current Situation

\[ \langle M', 0\nu \rangle \]

- NSM (Jastrow)
- \( (R)QRPA \) (Jastrow, UCOM)

Element Numbers:
- Ge 76
- Se 82
- Zr 96
- Mo 100
- Cd 116
- Te 128
- Te 130
- Te 136
- Xe 138
Current Situation

![Graph showing the current situation with data points for NSM (Jastrow) and (R)QRPA (Jastrow, UCOM). The graph includes isotopes such as Ge, Se, Zr, Mo, Cd, Te, Xe with corresponding values.](image-url)
Shell-Model Operators from First Principles

Let

\[ P = \sum_{i \in \text{SM space}} |i\rangle \langle i| \]

\[ Q = \sum_{\text{other } i} |i\rangle \langle i| \]

Bloch-Horowitz Equations

\[ H_{\text{eff}}(E) = PHP + PHQ 1 E - QHQHP \]

\[ |\Psi_a\rangle = Z(1 + E_a - QHQH) P |\Psi_a\rangle \]

\[ H_{\text{eff}}(E_a) P |\Psi_a\rangle = E_a P |\Psi_a\rangle \]

\[ \langle \Psi_a | M | \Psi_b \rangle = \sqrt{\langle \Psi_a | P | \Psi_a \rangle \langle \Psi_b | P | \Psi_b \rangle} \]
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\[ H_{\text{eff}}(E_a) P |\Psi_a\rangle = E_a P |\Psi_a\rangle \]
\[ \frac{\langle \Psi_a | P M^{\text{eff}} P |\Psi_b\rangle}{\sqrt{\langle \Psi_a | P |\Psi_a\rangle \langle \Psi_b | P |\Psi_b\rangle}} = \langle \Psi_a | M |\Psi_b\rangle \]
Perturbative Expansion

Removes dependence of effective operators on energies and leads to diagrammatic series for matrix elements.

For Effective Hamiltonian:

\[ G = V + V V + \ldots \]
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For Effective Hamiltonian:

1. Define $G$ matrix by iterated sum over high-lying “two-particle” states:

$$G = V + VV + \ldots$$
Then expand in $G$ to get the interaction $H_{\text{eff}}$:
Effective Double-Beta Operator

By analogy,

1. Define a $\beta\beta$ operator that includes high-energy stuff:

\[ M_{\text{high}} = M + G \tilde{G} + M \tilde{G} + \tilde{G} M + \tilde{G} \]
Expand in $G$ to get full effective operator $\mathcal{M}_{\text{eff}}$:
Results for $^{82}$Se

Diagrams folded with shell model transition densities from Poves et al.
Results for $^{82}\text{Se}$

Diagrams folded with shell model transition densities from Poves et al.

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Full Matrix Element

- Bare: 3.78
- Without 3p-1h: 6.24
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- QRPA ($g_A = 1.25$): $\approx 5$
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Increase from ladders and $4p-2h$ (pairing... ) large, but mostly canceled by $3p-1h$. 
Short-Range Correlations

These come mainly from two particles in high-energy states, i.e. from nonperturbative ladder sum $\mathcal{M}_{\text{high}}$:

$$
\mathcal{M}_{\text{high}} = \mathcal{M} + \tilde{G} \mathcal{M} + \mathcal{M} \tilde{G} + \tilde{G}^{\ast} \mathcal{M}
$$

Results under reasonable control.
Short-Range Correlations for $^{82}$Se

\[ C_{\text{GT}} \quad (\text{fm}^{-1}) \]

- **Bare**
- **Microscopic correction**
- **Phenomenological Jastrow**

**No forbidden currents, form factors**

**With forbidden currents, form factors**

$r$ (fm)
Some Other Schemes

Variational Monte Carlo: Minimize

$$\mathcal{E}_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle}$$

with correlated trial wave function (schematic here):

$$|\Psi_T\rangle = \left[ 1 + \sum_{i<j<k} U_{ijk} \right] \left[ S \prod_{i<j} U_{ij} \right] |\Phi\rangle.$$  

$U$’s represent two- and three-body correlations, $|\Phi\rangle$ is a Slater determinant.
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- **Unitary Correlation Operator Method (UCOM):** For every operator \( \hat{O} \), let

\[ \hat{O} \longrightarrow C^\dagger \hat{O} C. \]

\( C \) chosen to mimic strong short-range repulsion in \( H \).
Renormalization at short distances

$^{82}\text{Se (no forbidden...)}$

- Bare
- Phenomenological Jastrow
- Ladder-diagram sum
- Unitary Correlator Operator
- Variational Monte Carlo

$C_0$ vs $r$ (fm)
Variational Monte Carlo More Carefully

Paired Nuclear Matter in Periodic Box

![Graph showing 3- or 4-body effect, apparently](image_url)
Variational Monte Carlo More Carefully

Paired Nuclear Matter in Periodic Box

3- or 4-body effect, apparently
Coming: Full Nonperturbative Renormalization

Three promising methods for long term

- Coupled clusters
- No-core shell model (NCSM)
- Similarity Renormalization Group (in medium)

Right now: light nuclei as laboratory

Full NCSM calculation of light p-shell nuclei (few nucleons outside $^4$He core). Use $^6$Be $\rightarrow ^6$He to get accurate effective p-shell interaction and operator for 2-body system. Test 2-body operator in systems with more nucleons. Get 3-body piece from $^7$Be $\rightarrow ^7$He, etc.
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- Similarity Renormalization Group (in medium)

Right now: light nuclei as laboratory

- Full NCSM calculation of light p-shell nuclei (few nucleons outside $^4$He core). Use $^6$Be $\rightarrow$ $^6$He to get accurate effective p-shell interaction and operator for 2-body system.
- Test 2-body operator in systems with more nucleons.
- Get 3-body piece from $^7$Be $\rightarrow$ $^7$He, etc.
In-Medium SRG

\[ H_s \equiv U(s)H U^\dagger(s) \rightarrow \frac{dH}{ds} = [\eta(s), H_s] \quad \text{with} \quad \eta(s) = \frac{dU}{ds} U^\dagger(s) \]
\[ H_s \equiv U(s)H U^\dagger(s) \rightarrow \frac{dH}{ds} = [\eta(s), H_s] \quad \text{with} \quad \eta(s) = \frac{dU}{ds}U^\dagger(s) \]

Different \( \eta \)'s do different things. The choice

\[ \eta(s) = PH_s P + QH_s Q \]

drives \( H_s \) to be block diagonal.
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**Problem:** Many-body operators generated by evolution.

Development in really early stages...
In-Medium SRG

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**Problem:** Many-body operators generated by evolution.

**Partial Solution:** Normal order wrst Hartree-Fock state.

Truncation at normal-ordered 2-body level builds in important 3-body effects.

Development in really early stages...
That’s all.
Appendix: Situation 5 Years Ago

This was embarrassing. Theorists took some steps.
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This was embarrassing. Theorists took some steps...
Fit QRPA to $2\nu$ Decay

\[
M^{2\nu} \text{ (MeV}^{-1}) \quad \text{vs} \quad g_{pp}
\]

- 9 levels
- 21 levels

\[
M^{0\nu}
\]
Fit QRPA to $2\nu$ Decay

Number of Calculations

$m_{\text{eff}}$ (eV) for lifetime $4 \times 10^{27}$ y

76 Ge

shell model
Fit QRPA to $2\nu$ Decay

Number of Calculations

-shell model

"consolidated" QRPA

76 Ge

$m_{\text{eff}}$ (eV) for lifetime $4 \times 10^{27}$ y
Fit QRPA to $2\nu$ Decay

Not perfect Fitting $2\nu$ worsens $\beta^+$ and $\beta^-$ from first intermediate $1^+$. 

"consolidated" QRPA
Shell-Model vs. QRPA After Fitting

Results can still differ by factor of 2 or more
Recent Measurement of Occupation Numbers

Vacancies

<table>
<thead>
<tr>
<th>Neutron Vacancy</th>
<th>Experiment</th>
<th>QRPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>76Ge</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>76Se</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>76Ge</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>76Se</td>
<td>7.9</td>
<td></td>
</tr>
</tbody>
</table>

For the neutrinoless double beta decay, the data shows significantly larger values than the calculations. For the 0
0
reaction type, the summed normalizations are also similar. Third, the summed vacancies in the QRPA results are in good agreement with the expected values of 8, 6, 8, and 6.

FIG. 3 (color online). The deduced neutron vacancies for these two nuclei. The lower part of the figure shows the differences in these occupations (expected to be 2.0), compared to those from the QRPA calculations of

\[ \text{Reference [10]} \]

While the QRPA results predict changes between the

\[ \text{Reference [7]} \]

What the consequences may be, of this disagreement in

\[ \text{Reference [5]} \]

...