Quantum Mechanics, Physics 531
Midterm Exam 2, due April 20, 2009 at 1:20 pm

Problem 1. *(20 points)* Compute

$$\langle(\Delta\hat{S}_i)^2\rangle = \langle\hat{S}_i^2\rangle - \langle\hat{S}_i\rangle^2$$

for $i = x, y, z$ for a spin-up state of a spin $S = 1/2$ particle. Using your result, check the generalized uncertainty relation

$$\langle\Delta\hat{A}^2\rangle\langle\Delta\hat{B}^2\rangle \geq \frac{1}{4}\left|\langle[\hat{A},\hat{B}]\rangle\right|^2$$

for all three choices of pairs of the $\hat{S}_i$ operators.

Problem 2. *(20 points)* The spin dependent Hamiltonian of an electron-positron pair in the presence of magnetic field along z direction can be written as

$$\hat{H} = A\hat{\mathbf{S}}^{(e)}\hat{\mathbf{S}}^{(p)} + B(\hat{S}_z^{(e)} - \hat{S}_z^{(p)}).$$

Suppose the spin function of the system is given by $|\uparrow_e\rangle \otimes |\downarrow_p\rangle$.

a) Is this an eigen function of the spin Hamiltonian $\hat{H}$ for $A = 0$? If it is, what is the eigen energy? If not, what is the expectation value of the spin Hamiltonian $\hat{H}$?

b) Same problem when $B = 0$, $A \neq 0$.

Problem 3. *(30 points)* Construct the matrix $\hat{L}_{\boldsymbol{n}}$ representing the projection of the angular momentum operator on direction $\boldsymbol{n} = \{\sin\theta\cos\varphi; \sin\theta\sin\varphi; \cos\theta\}$ in the basis of eigenstates of the angular momentum along z axis ($\theta = 0$) with total angular momentum $l = 1$. Find the eigenvalues and the normalized eigenvectors.

Problem 4. *(30 points)* The electrostatic potential of a screened positive charge in a medium has the form

$$V(r) = \frac{|e|}{4\pi\epsilon\epsilon_0}\frac{e^{-r/r_c}}{r},$$

where r_c is the screening radius. With a trial wave function $\psi(\boldsymbol{r}) = \exp(-r/b)/\sqrt{\pi b^3}$, estimate the binding energy of an electron (with charge $-|e|$) by the positive charge. Show that the bound state exists for $r_c \gg a$, where a is the Bohr radius. Does the bound state always exist for arbitrary small values of the screening radius?