Quantum Mechanics, Physics 531
Second trial for Midterm 1, due May 5, 2008

Consider a harmonic oscillator, described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(x), \quad U(x) = \frac{1}{2} m\omega^2 x^2.$$

The wave function at $t = 0$ is given by

$$\psi(x/\lambda_q, t=0) = \psi(\xi, t=0) = A(2\xi^2 - 2\xi - 1)\exp(-\xi^2/2), \quad \lambda_q = \sqrt{\frac{\hbar}{m\omega}}.$$

Question 1 (10 points).
Find the representation of $\psi(x/\lambda_q, t=0)$ in the basis of eigen states of the Hamiltonian, see Eq. (1) below. Calculate the normalization factor A.

Question 2. (10 points).
Write down the wave function $\psi(x/\lambda_q, t)$ at arbitrary time t.

Question 3. (15 points).
What is the average value of energy of the oscillator in state $|\psi(x,t)\rangle$? If the energy of an oscillator were measured, what values the measurement may give and with what probabilities?

Question 4. (20 points).
Calculate the average values of the coordinate, $\langle x(t)\rangle = \langle\psi(x,t)|\hat{x}|\psi(x,t)\rangle$, and the momentum, $\langle p(t)\rangle = \langle\psi(x,t)|\hat{p}|\psi(x,t)\rangle$.

Question 5. (20 points).
Evaluate $\langle\psi(x,t)|\hat{x}^2|\psi(x,t)\rangle$ and $\langle\psi(x,t)|\hat{p}^2|\psi(x,t)\rangle$.

Question 6. (15 points).
Check that the result of the previous question is consistent with the Ehrenfest's theorem:

$$\frac{d\langle x(t)\rangle}{dt} = \frac{\langle p(t)\rangle}{m}, \quad \frac{d\langle p(t)\rangle}{dt} = \Big\langle -\frac{\partial U(x)}{\partial x}\Big\rangle = -m\omega^2\langle x(t)\rangle.$$

Supplementary information: The first few eigen functions of the harmonic oscillator are given by

$$\begin{aligned}
\psi_0(\xi) &= \frac{1}{\sqrt[4]{\pi}}\exp(-\xi^2/2); \\
\psi_1(\xi) &= \frac{\sqrt{2}}{\sqrt[4]{\pi}}\xi\exp(-\xi^2/2); \\
\psi_2(\xi) &= \frac{2\xi^2 - 1}{\sqrt{2}\sqrt[4]{\pi}}\exp(-\xi^2/2)
\end{aligned} \tag{1}$$