**Quantum Mechanics, Physics 531**
**Midterm Exam 2, due April 21, 2008 at 1:20 pm**

**Problem 1.** *(20 points)* Find the commutator $\hat{K}_0 = [\hat{K}_+; \hat{K}_-]/2$ of the operators $\hat{K}_\pm = \hat{a}_\pm \hat{a}_\pm/2$, where the operators $\hat{a}_\pm$ are the raising and lowering operators of a harmonic oscillator, their commutator is $[\hat{a}_-; \hat{a}_+] = 1$.

Calculate the commutation relations $[\hat{K}_\pm; \hat{K}_0]$.

**Problem 2.** *(30 points)* Find eigen energies of a particle moving in a sector of angle $\theta$ of a plane. The potential in this sector is parabolic, $U(r) = m\omega^2 r^2/2$, and $r = 0$ is the corner of the sector, see figure below. The potential in the rest of the plane is infinitely high. Due to infinite potential at the edges of the sector, the wave function must vanish there, and can be written in the polar coordinates as $\psi(r, \vartheta) = \phi(r)\sin(\pi k\vartheta/\theta)$ with integer $k$.

**Problem 3.** *(40 points)* Using the semiclassical Borh-Sommerfeld's quantization rule, calculate the energy difference between two adjacent levels of a weakly anharmonic oscillator, described by the Hamiltonian

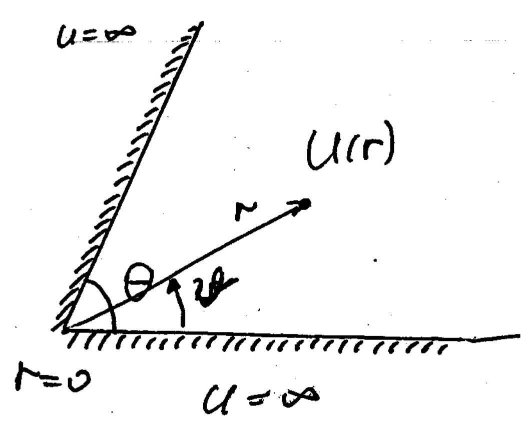
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar\omega}{2}\left[\frac{x^2}{\lambda_q^2} + \epsilon^2\frac{x^4}{\lambda_q^4}\right],$$

where $\lambda_q = \sqrt{\hbar/m\omega}$ and $\epsilon \ll 1$. The following integral is helpful

$$\int_{-1}^{+1} \frac{d\xi}{\sqrt{1 - (\xi^2 + a^2\xi^4)/(1 + a^2)}} = 2K\left(\frac{-a^2}{1 + a^2}\right) \approx \pi - \frac{\pi}{4}a^2,$$

where $K(x)$ is an elliptic function and the last approximation is valid for $a \ll 1$.

Compare your answer with the result of the first order perturbation theory.




updated Apr 17, 2008