Quantum Mechanics, Physics 531
Homework Assignment 2, due February 25, 2008

Problem 1. Problem 2.31.

Problem 2. Problem 2.34.

Problem 3. Problem 2.38.

Problem 4. A particle of mass m in one dimension is bound to a fixed center by an attractive $\delta-$function potential $V_\alpha(x) = -\alpha\delta(x)$ with $\alpha > 0$. A wave function is given by $\psi_\alpha(x) = \sqrt{\alpha m}/\hbar \exp(-m\alpha|x|/\hbar^2)$. At $t = 0$ the potential is suddenly changed to $V_\beta(x) = -\beta\delta(x)$ with $\beta > 0$. Calculate

$$\chi_{\alpha\beta} = \int_{-\infty}^{+\infty} \psi_\alpha(x)\psi_\beta(x)dx.$$

Show that $|\chi_{\alpha\beta}|^2 \leq 1$ and provide the interpretation of quantities $|\chi_{\alpha\beta}|^2$ and $1 - |\chi_{\alpha\beta}|^2$.

Problem 5. Determine the energy levels of bound states in the potential

$$U(x) = \begin{cases} U_l, & x < 0; \\ 0, & 0 < x < L; \\ U_r, & x > L. \end{cases}$$

with $U_{l,r} > 0$. Do not solve the corresponding transcendental equation, just derive it.

Show that for $U_l \neq U_r$ there are no bound states in a sufficiently narrow well with $L < L^*$. Estimate L^*.

Problem 6. Using the uncertainty relation for coordinate and momentum, determine the lower limit for the possible values of the energy of a quantum harmonic oscillator.